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Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

L. H. DONNELL, *Editor*

T. VON KÁRMÁN, S. TIMOSHENKO, *Editorial Advisers*

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Abbreviations of units follow the standard of Abbreviations for Scientific and Engineering Terms of the Am. Standards Assoc. Examples: psi (pounds per square inch); cps (cycles per second); mph (miles per hour).

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Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

September 1948 Vol. 1, No. 9

General Kinematics, Statics, Dynamics

(See also Rev. 1421)

1305. **E. Pawelka, "100 exercises in mechanics (100 Übungen aus der Mechanik),"** Springer-Verlag, Wien, Austria, 1948, Paper, 5.3 × 8.2 in., 187 pp., 154 figs., $2.60.

The problems of this collection are strongly realistic, although never numerical, and rather more involved than the ones encountered in most texts in engineering mechanics. They approximate the routine problems likely to be tackled by an industrial engineer. Some of them are standard and general enough no be regarded as theoretical. There are five groups of them: kinematics; statics; kinetics; beams and plates, etc.; elastic vibrations and waves. All problems are solved.

D. R. Mazkevich, USA

1306. **W. S. Hemp, "Note on the dynamics of a slightly deformable body,"** *Coll. Aero. Cranfield Rep.*, no. 5, Feb. 1947 pp. 1–5.

The paper formulates general equations of motion of a slightly deformable body. The motion is resolved into three parts: translation, rotation, and vibration, the latter two being coupled except for small angular velocities.

Henry A. Lepper, Jr., USA

1307. **H. Dietz, "The solution of the problem of a six rod-joint system by the method of dual mapping (Die Lösung des Sechsstabanschlusses mit der Methode der dualen Abbildung),"** *Ingen.-Arch.*, 1947, vol. 16, no. 1, pp. 14–38.

By means of R. Sauer's method of "dual mapping" [*Z. angew. Math. Mech.*, 1940, vol. 20, p. 174] a spatial system of forces acting on a rigid body is replaced by two new systems. One of them is plane and it's equilibrium expresses the vanishing of the resultant force of the spatial system; the other consists of parallel forces (preferably perpendicular to the plane above) and it's equilibrium is equivalent to the vanishing of the resultant moment. Each of the systems can be treated graphically. Procedures are developed for special cases.

The paper is quite elementary and deserves the attention of authors of larger texts in engineering statics.

R. G. Boiten, Holland

Gyroscopics, Governors, Servos

1308. **G. Kron, "Tensorial analysis of control systems,"** *J. appl. Mech.*, June 1948, vol. 15, pp. 107–124.

This paper shows by an example that it is possible to set up once for all the equations of each unit or stage of a servomechanism quite independently of the presence of the other units. Then it is possible to combine these equations in the same manner in which their physical equivalents are combined, thus simplifying study of the many combinations possible. This procedure is quite automatic and requires no analytical or physical reasoning. The examples considered are the various types of speed, pressure, and position-governing systems used in central-station work, each system containing interchangeable components. The paper confines itself to the routine establishment of the equations of small oscillations for a given servomechanism, and does not undertake their solution.

The equations of small oscillations about a state of steady motion may usually be represented by a set of linear algebraic equations $Z_{\alpha\beta}\Delta x^{\beta} = \Delta f_{\alpha}$. When a complex dynamical system is given, it is possible to establish the resultant equations by first removing all interconnections between the various amplification stages, and finding the impedance matrix $Z_{\alpha\beta}$ of each isolated stage. The effect of their interconnection is representable by a matrix of transformation $C_{\alpha}{}^{\prime\alpha}$. The impedance matrix $Z_{\alpha'\beta'}$ of the resultant system may be found without any physical analysis by a simple routine matrix manipulation involving $Z_{\alpha\beta}$, $C_{\alpha}{}^{\prime\alpha}$, its partial derivatives $\partial C_{\alpha}{}^{\prime\alpha}/\partial x^{\alpha}$, and the steady-state force vector f_{α}. (In oscillatory problems the law of transformation of the impedance matrix $Z_{\alpha\beta}$ is not that of a tensor.)

The method is illustrated by setting up the differential equations of various types of speed, pressure, and position-governing systems used in central-station work. The mathematical background needed is only that of addition and multiplication of matrices.

Alexander Mendelson, USA

1309. **N. A. Kartvelishvili, "On the conditions of quality of automatic control"** (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, July 1, 1948, vol. 61, pp. 21–23.

General equations for the transfer process are given in terms of Heaviside's operator, $p = \eta + j\omega$. The quality of regulation is prescribed by setting values for η_0 and ω_0 and requiring that the polynomial, $F(p)$, contained in the solution, shall have no zeros outside a horizontal rectangular strip of width $-\omega_0$ to $+\omega_0$ extending from $-\eta_0$ to $-\infty$.

Walter W. Soroka, USA

Vibrations, Balancing

(See also Revs. 1305, 1300, 1313, 1338)

Wave Motion, Impact

(See also Revs. 1305, 1315, 1423, 1437)

1310. **M. M. Freedman, Diffraction of a plane elastic wave with reference to a semi-infinite rectilinear rigidly fixed slit"** (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, June 1, 1948, vol. 60, pp. 1145–1148.

The note contains a solution of the two-dimensional problem of diffraction of a plane (longitudinal or transverse) elastic wave by a semi-infinite rigidly fixed rectilinear slit. The solution is obtained by making use of the functionally invariant solutions of the wave equation deduced by V. I. Smirnov and S. L. Sobolev, and by applying the methods of N. I. Muskhelishvili's monograph on *Singular Integral Equations*, Moscow (1946).

I. S. Sokolnikoff, USA

Elasticity Theory

(See also Revs. 1322, 1323, 1324, 1333, 1363)

1311. L. A. Galin, "An estimate for the displacement in spatial contact problems of the theory of elasticity" (in Russian), *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, May-June 1948, vol. 12, pp. 241–250.

This paper is concerned with the problem of contact of a rigid punch with an elastic half space. In Sec. 1 the base of the punch is assumed to be bounded by the surface $z = f(x, y)$, and the section of the punch made by the plane $z = 0$ is an ellipse D. Thus (x, y) is the displacement of the punch in the z-direction. The punch is acted on by the force P directed along the z-axis and by moments M_x and M_y with respect to the x and y-axes. The state of stress is determined by a harmonic function $\phi(x, y, z)$ such that $\phi = f(x, y)$ in D, $\partial\phi/\partial z = 0$ outside D, and $\phi(x, y, z) \longrightarrow 0$ as $\sqrt{x^2 + y^2 + z^2} \longrightarrow \infty$. The author determines ϕ as a series of Lamé's functions, and calculates the displacement in the z-direction and the rotation of the punch relative to the x and y-axes in terms of P, M_x, and M_y.

In Sec. 2 the base of the punch is assumed to be plane and the cross section enclosing the area A of contact bounded by an arbitrary curve L. Let a be the major semiaxis and e the eccentricity of an ellipse circumscribing L, then for a given displacement δ of the punch, the impressed force P is bounded by

$$\frac{2\delta E}{1 - \delta^2}\sqrt{\frac{A}{\pi}} < P < \frac{\delta E}{1 - \delta^2}\frac{\pi a}{F(\pi/2, a)}$$

where E is Young's modulus, δ is Poisson's ratio, and $F(\pi/2, a)$ is the complete elliptic integral of the first kind.

I. S. Sokolnikoff, USA

1312. S. Ghosh, "On the flexure of an isotropic elastic cylinder" (in English), *Bull. Calcutta math. Soc.*, Mar. 1947, vol. 39, pp. 1–14.

In this paper the Schwartz techniques of conformal mapping are applied by the author to the general problem of St. Venant flexure of an isotropic elastic cylindrical bar. This procedure had previously been applied by Muskhelisl vili to the special case of pure St. Venant torsion. The author succeeds in deriving general expressions for the shearing stresses at any point of the cross section of the cylinder in terms of definite integrals involving the mapping function which carries the given boundary of the cross section into the unit circle.

The method is applied to the special problem of a cylinder whose cross section is bounded by a cardioid and the results are compared with those found by other investigators.

Frederick Seitz, USA

1313. M. I. Rosovski, "Application of integral and integro-differential equations to the study of deformation processes of real materials" (in Russian), *Bull. Acad. Sci. USSR Ser. tech. Sci. (Izv. Akad. Nauk SSSR Ser. tekh. Nauk)*, May 1948, no. 5, pp. 601–622.

Stress-strain relations in real bodies should contain experimental time-dependent quantities (aftereffect coefficients). The author takes as basis the Boltzmann-Volterra equations which express stresses in terms of strains and aftereffect, and solves them for the component deformations. He works out several particular cases: (1) A prismatic bar with given load or with given strain; (2) longitudinal vibrations of a prismatic bar, free or forced; (3) transverse vibrations of a compressed bar; (4) torsional vibrations; (5) a circular cylinder being pressed into a base, the material of the cylinder following Hooke's law, and that of the base the Boltzmann-Volterra equations.

All these problems are solved by taking into consideration the influence of time on the stabilization of stresses and strains. Owing to the character of the B.-V. equations, the author is forced to devote most of his work to the solution of integral and integro-differential equations. Some of his results are compared with those of other Russian papers and good agreement is found.

Witold Wierzbicki, Poland

1314. P. M. Riz, "On the asymptotic integration of the equations of the theory of elasticity with applications to a plate and a disk of variable thickness" (in Russian), *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, May-June 1948, vol. 12, pp. 349–352.

This note contains a sketch of the method of approximate integration of the three-dimensional equilibrium problem of elasticity for "thin" bodies. The bodies under consideration are symmetrical with respect to the xy-plane and are determined by the equation of the form $y = \beta f(x, z)$, where β is a small thickness parameter. The displacements u, v, and w are assumed to have expansions of the type $u = u_0 + \sum_{n=1}^{\infty} u_n \beta^n$. If one introduces the variable $\eta = \beta y$, the equations of Lamé in terms of x, y, and z assume the form typified by

$$\mu\left(\nabla^2 u + \frac{1}{\beta^2}\frac{\partial^2 u}{\partial \eta^2}\right) + (\lambda + \mu)\frac{\partial \Lambda}{\partial x} + \frac{1}{\beta}\frac{\partial^2 v}{\partial x \eta} = X$$

where $\nabla^2 = (\partial^2/\partial x^2) + (\partial^2/\partial_z{}^2)$ and $\Lambda = (\partial u/\partial x) + (\partial w/\partial z)$

The substitution of expansions for u, v, and w in such equations and the usual argument of the method of small parameters yields an infinite system of differential equations for the determination of u_i, v_i, w_i. The boundary conditions for the displacements are determined in a similar way.

As an illustration of the method, the problem of deformation of a disk whose surface is given by $y = \beta f(x^2 + z^2)$ and subjected to radial body forces is discussed. The results coincide with known equations for displacements in rotating turbine disks.

I. S. Sokolnikoff, USA

1315. L. S. Leibenzon, "Course in the theory of elasticity" (in Russian) 2nd ed., OGIZ, Moscow-Leningrad, 1947, 464 pp.

This revised edition of a book published in 1941 and based on an introductory course in the theory of elasticity contains three new chapters: Theory of Bending of Thin Plates, Equations of Plastic Deformation, and Variational Methods of Solution of Elastic Problems. The presentation is clear and concise, and the level of the book approximates that of R. V. Southwell's *Theory of Elasticity*. The author avoids vector and tensor calculus and follows Love's symbolism.

The first five chapters contain the analysis of deformed and stressed states and include the derivation of the equilibrium equations. A noteworthy feature is the formulation of the equilibrium equation in terms of Galerkin's (1931) and Neuber-Papkovitch's (1934) stress function.

Chapters 6 and 7 deal with the classical three-dimensional problems of elastic equilibrium of the sphere and the problems of Boussinesq and Hertz. Chapters 8, 9, and 10 are concerned with two-dimensional elastic problems, plane stress and strain, and Saint Venant's theory of flexure and torsion. A brief account of the use of the complex variable theory in plane problems, based on the work of Kolossoff and Muskhelishvili, is included. Chapter 11 deals with variational principles of elasticity and establishes

the connection of Castigliano's principle with Saint Venant's compatibility conditions (first pointed out by Southwell in 1936). The thermoelastic equations are given in Chapter 12, and the rudiments of thin plate theory are presented in less than thirty pages in Chapter 13. The inclusion of a brief account of the theory of plasticity (Chapter 14) in a book on elasticity is a welcome departure from usual practice. This sketches the contributions to the theory by von Mises, Hencky, Prandtl, Reiss, and Prager.

Chapter 15 gives a summary of basic facts about the propagation of elastic waves, and the concluding Chapter 16 furnishes a formal introduction to direct methods of solution of variational problems in elasticity. The book can be warmly recommended to theoretically inclined engineers and to beginning students in applied mathematics. I. S. Sokolnikoff, USA

1316. **A. E. Green and T. J. Willmore, "Three-dimensional stress systems in isotropic plates. II,"** *Proc. roy. Soc. Lond. Ser. A*, May 27, 1948, vol. 193, pp. 229–248.

This paper is a continuation of one of the authors' investigations concerning the accuracy of the usually accepted theory of generalized plane stress. In a previous paper [A. E. Green, *Phil. Trans. roy. Soc. Lond. Ser. A*, 1948, vol. 240, p. 561] the need for exact three-dimensional solutions upon which to base the evaluation of the accuracy of the generalized plane stress theory was pointed out, and the three-dimensional problem of stress concentration about a circular cylindrical hole was solved.

In the present paper the problem of an isolated force uniformly distributed through a plate of finite thickness and acting in the plane of the plate is solved as a three-dimensional problem. The solution of Lamé's equations is obtained in integral form. This exact solution is then compared with the generalized plane stress solution.

The accuracy of the approximate plane stress solution is found to be generally good except near the concentrated load, where the exact solution has doubtful physical meaning. The accuracy is found to depend upon Poisson's ratio, being best for moderate values and deteriorating a little as Poisson's ratio approaches the value 0.5. Near the load the normal stress in the direction perpendicular to the plate is found to be of the same order of magnitude as the other stresses. This stress is neglected in the approximate plane stress solution. Paul F. Chenea, USA

1317. **S. Ghosh, On a new function-theoretic method of solving the torsion problem for some boundaries"** (in English), *Bull. Calcutta Math. Soc.*, Sept. 1947, vol. 39, pp. 107–112.

A method is presented of solving the torsion problem for beams where a portion of the boundary of the cross section consists of a straight line. Complex variables are used.

In the solution, it is necessary to find the function which maps conformally the cross section on the upper half of a unit circle, the straight portion mapping on the bounding diameter. Formulas are derived for finding the complex torsion function and finally the torsional rigidity in terms of the mapping function.

Two examples are given; one for a semicircular cross section; the other for a semiloop of the lemniscate of Bernoulli.

Gerald Pickett, USA

1318. **M. I. Rosovski, "Plane deformation with elastic aftereffect and thermal stress"** (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, Nov. 21, 1947, vol. 58, pp. 999–1002.

The author generalizes the Hooke-Volterra stress-strain relations in a long homogeneous isotropic elastic cylinder by including terms accounting for the thermal stresses. He is led to an integro-differential equation for the displacements, which is solved in terms of the resolvents of certain kernels.

Courtesy of Mathematical Reviews I. S. Sokolnikoff, USA

1319. **D. C. Gilles, "The use of interlacing nets for the application of relaxation methods to problems involving two dependent variables,"** *Proc. roy. Soc. Lond. Ser. A*, July 2, 1948, vol. 193, pp. 407–433.

In the solution by relaxation methods of partial differential equations involving two dependent variables, it has been customary to find the values of all such variables at the nodes of one net. By considering the relative accuracy of the finite difference approximations to the individual terms of the equations, and for other reasons, it is suggested that values of some of the variables be determined at the centers of the meshes, which amounts to using two interlacing nets.

In the case of problems to which this idea can be applied, the use of a system of two conjugate equations with two variables is to be recommended, instead of a single equation resulting from the elimination of one variable of the system.

The simplest example of the application of the above idea would probably consist in the use of the Cauchy-Riemann system instead of the Laplace equation. This method is used by the author in the following problems: (1) Plane strain with boundary displacements given; (2) stresses in a solid of revolution; (3) the conjugate plane-harmonic functions; (4) the quasi-plane-harmonic equation. M. Bricas, Greece

Experimental Stress Analysis

1320. **M. M. Frocht, "On the removal of time stresses in three-dimensional photoelasticity,"** *Proc. Soc. exp. Stress Anal.*, 1948, vol. 5, no. 2, pp. 9–13.

The author reports that time stresses in thin slices freshly cut from frozen-stress models tend to disappear gradually with time leaving a specimen of BT 61893 Bakelite essentially free of time stress after about three days.

This observation goes far toward eliminating the serious time stress problem in the use of the above material for three-dimensional stress studies by the frozen-stress technique. Examples are also shown of the reduction of time stresses by removing the compressively stressed surface skin and the rapidity with which these stresses reform.

The theory advanced for the first observation and the explanation that "owing to the absence of equilibrium (of time stresses on one plane of the slice) release of the time stresses in a thin plate is a logical expectation," does not seem very convincing.

William N. Findley, USA

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 1332, 1334, 1344)

Plates, Disks, Shells, Membranes

(See also Revs. 1305, 1315, 1326, 1327, 1328, 1329, 1330, 1353)

1321. **A. G. Holms and J. E. Jenkins, "Effect of strength and ductility on burst characteristics of rotating disks,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1667, July 1948, pp. 1–52.

This paper records the results of tests to determine bursting speeds of rotating disks made of various materials having widely

varying strength and ductility properties. The disks, of 10-in. outside diameter and of approximately $^3/_8$-in. thickness, were driven by an air turbine. Tests were made both on solid disks and on disks with small ($^1/_{16}$-in.) and large ($1^1/_2$-in.) diameter holes. Materials tested included SAE 1078 and SAE 1035 steels, stainless steel, beryllium copper, brass and aluminum and nickel-base alloys. Ductility in the materials used varied as indicated by a range of tensile elongation values from 3.4 to 52.8 per cent. The solid disks and those with small holes were driven by means of a small hub while the disks with large holes were driven by means of a "tulip petal" arrangement.

In most cases, failures were of the brittle or cleavage type, at 90 deg to the plane of the disk, with the latter splitting into a number of fragments. The most ductile disks, however, failed at angles of 45 deg to the plane of the disk. Some disks with intermediate values of ductility showed both types of failure.

In general, it was found that for both solid disks and disks with holes, the disk strength (in terms of speed necessary to cause fracture) increased with tensile strength regardless of ductility. Introduction of a large hole reduced the strength approximately in proportion to the amount of material removed. A slight reduction in strength was also obtained for disks with small holes ($^1/_{16}$-in. diam) but nowhere near the 50 per cent reduction called for on the basis of the elastic-stress theory.

In the case of the solid disks, the ratio of the calculated elastic stress at failure to tensile strength ranged between the value of 1.0 predicted by the elastic-stress theory and the value of 1.24 predicted by the average-stress theory. For disks with holes the values of this ratio were, in general, somewhat below those predicted on the basis of the average-stress theory, but considerably above those predicted from the elastic-stress theory. It should be noted here that neither of these theories takes into account stress increases due to expansion in disk diameter or reduction of disk thickness at fracture.

Tests on disks made of a very hard material (Rockwell C 60) of low ductility showed a reduction in strength around 30–36 per per cent due to the presence of fine grinding cracks.

A. M. Wahl, USA

1322. L. A. Galin, "On the pressure of a solid body on a plate" (in Russian), *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, May-June 1948, vol. 12, pp. 345–348.

A rigid body whose surface is determined by the equation $z = Ax^2 + By^2$ is brought in contact with a thin clamped circular elastic plate of radius R. The point of initial contact is the center of the plate $x = 0$, $y = 0$. If the force P, acting along the z-axis, is applied to the body, what is the shape of the area of contact?

It is shown that the area of contact is an ellipse, provided that the linear dimensions of the area of contact are small compared with R and with the radii of curvature of the surface. The situation in this case is similar to that occurring in the problem of contact of a rigid ellipsoid with an elastic half space considered by H. Hertz.

I. S. Sokolnikoff, USA

1323. H. Hencky, "On the calculation of plane plates with consideration of shear stresses (Über die Berücksichtigung der Schubverzerrung in ebenen Platten)," *Ingen.-Arch.*, 1947, vol. 16, no. 1, pp. 72–76.

In this paper the author deals with the problem of determining the deformations of flat plates. In contrast to the classical plate theory where the influence of the transverse shearing stress is neglected, the mean values of these stress components are taken into account. The assumption of zero resultant transverse normal stresses is retained.

Choosing the Cartesian co-ordinate system so that the xy-plane coincides with the median plane of the plate, the displacement field is assumed in the form $u = -z/h\psi(x, y)$, $v = -z/h\chi(x, y)$, $w = \varphi(x, y)$ where h is the plate thickness and the initially arbitrary functions ψ, χ, φ are subsequently determined on the basis of the principle of virtual work. There result three partial differential equations for ψ, χ, and φ.

The author demonstrates that the new system of equations reduces to the classical plate equation if h is small compared to the other dimensions of the plate.

Sven T. A. Ödman, Sweden

1324. F. Rellich, "The boundary conditions of Airy's stress function for given displacements at the boundary (Die Randbedingungen der Airyschen Spannungsfunktion bei vorgegebenen Randverschiebungen)," *Z. angew. Math. Mech.*, Apr. 1947, vol. 25-27, pp. 13–17.

The stresses and displacements of a plate, the deflection (w) of which is comparable with its thickness (h), are governed by the two simultaneous differential equations

$$\Delta\Delta\Phi = E(w^2_{xy} - w_{xx}w_{yy})$$

$$\frac{Eh^2}{12(1-\nu^2)}\Delta\Delta w = \Phi_{yy}w_{xx} - 2\Phi_{xy}w_{xy} + \Phi_{xx}w_{yy}$$

[compare von Kármán, *Enzykl. Math. Wiss.*, vol. 4, art. 27, p. 350]. The displacements $u(x, y)$ and $v(x, y)$ are related to the stress function Φ and w by the equations

$$u_x = (1/E)(\Phi_{yy} - \nu\Phi_{xx}) - w_x^2/2$$

$$v_y = (1/E)(\Phi_{xx} - \nu\Phi_{yy}) - w_y^2/2$$

$$u_y + v_x = \frac{2(1+\nu)}{E}\Phi_{xy} - w_x w_y$$

The membrane stresses φ_x, φ_y, τ_{xy} are defined by

$$\varphi_x = \Phi_{yy},\ \varphi_y = \Phi_{xx},\ \tau_{xy} = -\Phi_{xy}$$

With prescribed stresses at the boundary no difficulties arise, the boundary conditions for Φ being the same as in the linearized theory [Biezeno and Grammel, "Technische dynamik," p. 99]. The problem becomes more intricate when the boundary conditions relate to the displacements u and v, but here too these conditions have to be transformed into conditions for Φ and w. This is done in the present paper.

C. B. Biezeno, Holland

1325. S. G. Lehnickiĭ, "The bending of a rectangular orthotropic plate resting on parallel rigid ribs" (in Russian), *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, May-June 1948, vol. 12, pp 339–344.

This note deals with the formulation of the boundary value problem for small deflections of thin rectangular orthotropic elastic plates reinforced by rigid ribs parallel to one of the sides of the plate. The load is normal to the middle plane, which is regarded as one of the planes of elastic symmetry. The other planes of elastic symmetry are parallel to the sides of the plate.

It is shown that the problem is equivalent to that of the deflection of a beam resting on an elastic foundation and subjected to tensile stresses along its axis. The methods of this paper are based, to a large extent, on the author's book [*Anisotropic Plates*, Gostehizdat, 1947]. For the solution of an analogous problem for an isotropic plate see A. S. Lokshin [*Appl. Math. Mech. (Prikl. Mat. Mekh.)*, 1935, vol. 2, no. 2] and A. P. Filippov [*Appl. Math. Mech. (Prikl. Mat. Mekh.)*, 1937, vol. 1, no. 2].

I. S. Sokolnikoff, USA

Buckling Problems

1326. N. J. Hoff, B. A. Boley, and S. V. Nardo, "The inward bulge-type buckling of monocoque cylinders. IV—Experimental investigation of cylinders subjected to pure bending," *Nat. adv. Comm. Aero. tech. Note*, no. 1499, Sept. 1948, pp. 1–73.

Eighteen 24S-T Alclad cylinders of 20-in. diam, stiffened with longitudinal stringers and transverse rings, were tested in pure bending. These tests were part of an extended investigation of general instability, a form of buckling involving both stringers and rings.

Previous work which is reported in the references had indicated that general instability could be expected when a certain parameter, involving the dimensions and relative stiffnesses of the components, exceeded a critical value. The present tests showed that a transition range apparently exists in which either the general or "panel" type of instability can be expected. The critical values of the parameter applicable to the specimens tested were determined, but the authors note that the relation found has not been substantiated for structures of larger size.

F. R. Shanley, USA

1327. E. W. Kuenzi and C. B. Norris, "Torsional buckling of longitudinally stiffened, thin-walled, plywood cylinders," *For. Prod. Lab. Rep.*, no. 1563, Jan. 1948, pp. 1–12.

Plywood cylinders were tested, about 0.04 in. thick, curved to a 9-in. radius, and with from 11 to 28 longitudinal stiffeners from $^1/_{32}$ to $^1/_2$ in. thick glued to the inner surface.

The results of torsion tests showed that the buckling stress of the portion of the shell between the stiffeners was about 85 per cent of the buckling stress of a complete unstiffened cylinder of the same curvature and thickness. The theoretical buckling torque of an unstiffened cylinder of the same weight and curvature as a stiffened cylinder was greater than that found for the stiffened cylinder.

T. J. Dolan, USA

1328. E. W. Kuenzi, "Effect of length on the buckling stresses of thin-walled, plywood cylinders in axial compression," *For. Prod. Lab. Rep.*, no. 1514, Mar. 1948, pp. 1–23.

This paper is a progress report on tests performed on short, thin-walled, plywood cylinders, of lengths shorter than the diameters, loaded by axial compression. The report presents the test data and empirical curves to be used in the design of short plywood cylinders with the face-grain direction axial or circumferential. If the face-grain direction is at 45 deg the buckling constant was found to be the average of those for 0 deg and 90 deg. Test results for longer cylinders were published previously [*For. Prod. Lab. Rep.*, nos. 1322 and 1322A]. Andrew Brodsky, USA

1329. G. H. Handelman and W. Prager, "Plastic buckling of a rectangular plate under edge thrusts," *Nat. adv. Comm. Aero. tech. Note*, no. 1530, Aug. 1948, pp. 1–97.

Equations for the strain increments in a material due to increases in stress in the plastic region, starting from a state of uniaxial compression, are given. In developing the equations it is assumed that (1) plastic deformations do not involve a change of volume, (2) plastic deformations occur only when the sign of the increment in work which the existing stresses do on the change of shape is positive, and (3) plastic increments in strain decrease continuously to zero in approaching the boundary of the region of plastic action. On the basis of these assumptions it is shown that all the empirical information necessary for the theoretical determination of the plastic buckling stress of a rectangular plate under edge thrusts can be obtained from the compressive stress-strain curve.

The differential equations for the plastic buckling of a simply compressed plate are derived, and their solutions are given for the case of plates simply supported on the loaded edges, and having either simple support or no support on the remaining edges. The results are compared with those of A. A. Ilyushin [*Appl. Math. Mech.* (*Prikl. Mat. Mekh.*), 1944, vol. 8, p. 337] and with those of S. Timoshenko [*Elastic Stability*, McGraw-Hill, 1936, p. 387]. In a particular example the new method gives buckling stresses 7 per cent higher than Timoshenko's method and 3 per cent higher than Ilyushin's.

The present theory of plastic buckling is distinguished from another recent theory of plastic buckling by Elbridge Z. Stowell [*Nat. adv. Comm. Aero. tech. Note*, no. 1556, Apr. 1948] by two main considerations. First, the present theory uses new stress-strain relations derived in the paper while Stowell uses those previously used by Ilyushin; and second, the present theory assumes the existence of a neutral surface separating the region of plastic action from the region of nonplastic action, while Stowell follows F. R. Shanley [*J. aero. Sci.*, May 1947, vol. 14, p. 261] by assuming that in plates, as well as in columns, unloading does not occur during the early stages of buckling, and that the plate therefore remains in the purely plastic state during buckling.

Samuel Levy, USA

1330. C. B. Norris, W. J. Kommers, and P. F. McKinnon, "Critical buckling strength of stiffened flat plywood plates in compression and shear—closely spaced stiffeners," *For. Prod. Lab. Rep.*, no. 1800, Feb. 1948, pp. 1–38.

Equations previously derived by the authors for the critical stresses in flat plywood plates subjected to edgewise compression or shear were applied to plywood stiffened with a multiplicity of parallel stiffeners. It was found that these equations apply with reasonable accuracy if the over-all elastic properties of the stiffened plates are used. Methods were derived for the computation of the over-all elastic properties of multiply stiffened plywood plates from the elastic properties of the plywood and the stiffeners, and the dimensions and spacing of the stiffeners.

R. L. Bisplinghoff, USA

1331. E. Z. Stowell, "Critical shear stress of an infinitely long plate in the plastic region," *Nat. adv. Comm. Aero. tech. Note*, no. 1681, Aug. 1948, pp. 1–19.

In a previous paper (see Rev. 805, APPLIED MECHANICS REVIEWS, May 1948) the author derived plasticity reduction factors η to be used in formulas for elastic plate buckling stress under several conditions, using an assumption that the plates remain everywhere in the plastic state during buckling. He here derives a similar factor for the case of a long plate under uniform shear. Some limited tests on shear buckling of aluminum-alloy plates in the plastic region tend to confirm the value of the reduction factor predicted by theory. Restraint against edge rotation is found to have little or no effect upon this reduction factor.

D. L. Holl, USA

1332. H. Ziegler, "Buckling of rods with oblique supports (Die Knickung des schief gelagerten Stabes)," *Schweiz. Bauztg.*, Feb. 14, 1948, vol. 66, pp. 87–89.

A thin prismatic rod of length l is loaded by a pair of compressive forces, acting at its ends. The z-axis of a rectangular coordinate system is directed along the length of the rod, the x and y-axis along the principal axes of inertia of an end cross section.

The end cross sections can rotate freely about the axis $z = 0$, $y = x \tan \alpha$ resp $z = 1$, $y = x \tan \alpha$.

The problem of the buckling of this rod gives rise to a differential equation for $u(z)$ (the projection of the elastic curve in the plane $y = 0$) and an analogous equation for $v(z)$ (the projection of the elastic curve in the plane $x = 0$). Because of the boundary conditions, u and v are coupled with each other; this makes the problem more general and more complicated than the elementary buckling problems of rods.

The lowest critical load is obtained in the classical way and given in dimensionless form in a diagram as a function of α and of the ratio of the principal moments of inertia.

W. L. Esmeijer, Holland

1333. L. Norzi, "Theory of elastic instability (Teoria dell 'instabilita elastica)," *Atti Accad. Torino*, 1945-1947 (issued in 1948), vols. 81-82, no. 1, pp. 253–263.

The author presents a formulation of the problem of elastic stability in terms of finite strains, from an analytic point of view using the energy method and a variational principle. He develops the usual necessary condition as well as the condition that for stability it is sufficient for the principal stresses to be everywhere positive (tensions) or at most, only the smallest in absolute value to be a compression. In particular, in the plane problem the smallest principal stress being zero in absolute value, the other two must be tensions everywhere. As a consequence of his theory of finite strain, he obtains a generalization of the equations of neutral equilibrium, and is able to express a general condition for instability by examining the work done by the externally impressed forces under a virtual displacement the components of which satisfy these equations and the conditions of constraint.

Edward Saibel, USA

1334. H. Templeton, "Approximate solutions for struts supported by an elastic foundation," *Aircr. Engng.*, July 1948, vol. 20, pp. 190–193.

The paper contains further applications of the energy method to the determination of critical compressive loads on struts subject to centrally or eccentrically applied axial forces, or initial deflections, with buckling resisted by an elastic medium continuous over the length of the strut. Derivation of the equations is concise but clear, and the illustrative numerical examples include helpful suggestions for minimizing computational labor.

The paper does not include a solution for the problem of the strut elastically supported at intervals—such as a longeron or stringer braced by transverse frames in a fuselage structure—though the reader of its introduction might expect one. Nor can the cases investigated be combined to effect such a solution. They serve, however, to demonstrate the applicability and relative simplicity of the energy method for solving elastic stability problems, and they are of value in that regard.

Joseph S. Newell, USA

1335. A. A. Beles, "On a buckling problem (Sur un problème de flambage)," *Bul. Polit. Bucarest*, Jan.-Apr. 1948, vol. 17, pp. 34–37.

The purpose of this short note is the qualitative examination of the possibility of buckling of a prismatic elastic bar placed in a closed vessel filled with a liquid under pressure.

Four cases are treated: A horizontal bar resting on the bottom of the vessel, a vertical bar built-in at the top, or at the bottom, and a horizontal bar simply supported at the ends. To solve them, the author states that a strained form of the bar will be unstable if the work done by the liquid pressures during the change of form surpasses the change in elastic strain energy of the bar. The work in question is equalto $V(p_b - p_l)(h_1 - h_2)$, where V is the volume of the bar, p_b and p_l the specific weights of bar and liquid, h_1 and h_2 the height of the center of gravity of the bar in its straight and curved forms respectively.

The reviewer does not agree with all of the author's conclusions. He feels that a horizontal bar is always incapable of buckling, whatever its conditions of support. This conclusion conflicts with that of the author. Ch. Massonnet, Belgium

Joints and Joining Methods

1336. H. H. Langdon and B. Fried, "Fatigue of gusseted joints," *Nat. adv. Comm. Aero. tech. Note*, no. 1514, Sept. 1948, pp. 1–40.

Fatigue tests were run on aluminum-alloy and heat-treated alloy-steel gussets to determine the effect of gusset edge finish on fatigue life and to compare riveted with spot-welded assemblies. Edge finish was found to have no effect on the fatigue life of either the 24S-T or the heat-treated alloy-steel (x 4130-AN-QQ-S685) gussets tested. The riveted 24S-T joints were found to have better fatigue characteristics than the spot-welded 24S-T joints. The results also indicate how stresses set up in the gusset during the riveting operation affect its fatigue life. Improvements in gusset design are indicated on the basis of these tests.

R. L. Bisplinghoff, USA

1337. W. J. Krefeld and E. C. Ingalls, "Residual stresses in a butt-welded structural I-beam," *Weld. Res. Suppl.*, Aug. 1948, vol. 13, pp. 417–420.

Supplementary tests were conducted by the authors to settle certain questions raised in a previous report ["An investigation of beams with butt-welded splices under impact," *Weld. Res. Suppl.*, July 1947, vol. 12, pp. 372–399], regarding the order of magnitude of residual stresses induced by the welding operation. The greatest stresses were produced in the weld metal, with an abrupt decrease in the immediately adjacent base metal where the stresses reversed in sign, indicating that the distribution of residual stresses in this butt-welded structural section was similar to that found in other investigations of flat plates. From the evidence at hand, it would seem that the small premature departure from linear elastic behavior observed in both the static and dynamic tests could be attributed largely to the influence of plastic yielding in the weld metal resulting from the superposition of residual welding stresses and stresses due to static or dynamic loads. Frederick K. Teichmann, USA

Structures

(See also Revs. 1307, 1326, 1334, 1348)

1338. A. Selberg, "Suspension bridges with cables fastened to the stiffening girders at the center of the bridge" (in English with summaries in German and French), *Publ. int. Ass. Bridge Struct. Engng.*, **1947**, vol. 8, pp. 195–205.

Although the analytical studies of suspension bridges began as early as 1823 (Navier), the treatment of the special problem of a suspension bridge in which the stiffening girder is tied to the cable at the center of the bridge is apparently new.

The author points out that tying or locking the stiffening girder to the cables are used in recent designs has the effect of reducing the moment and deflection in the stiffening girder, and tends to check the swinging of the bridge under the action of wind loads. The analysis assumes the stiffening girders to be straight with a

constant EI, and to be at the same elevation as the cables at the center of the bridge. It further assumes that the effect of the sloped position of the suspenders and the towers can be neglected.

An approximate correction to the theory of ordinary suspension bridges for the effect of the sloped position of suspenders is also presented. The application of the theory outlined in this paper is illustrated with an example and the results are compared with those obtained in an ordinary suspension bridge in which the stiffening girder is not locked to the cable of the bridge.

Karl Arnstein, USA

1339. **David Watstein, "Distribution of bond stress in concrete pull-out specimens,"** *J. Amer. Concr. Inst.*, May 1947, vol. 18, pp. 1041–1052.

The distribution of bond stresses and strain in the reinforcing bars was determined for various loads on pull-out specimens of five types of $^3/_4$-in. reinforcing bars (the new type reinforcing bars are not included). Specimens of both 8-in. and 12-in. embedment were tested. Strain in the bars was measured by 2-in. Tuckerman gages attached to strain-transfer devices operating through precast holes in the concrete. The bond stress and slip between the concrete and the bars was estimated from the measured strains in the bars. It was found that: (1) The bond stress varied less along the 8-in. bars than along the 12-in. bars; (2) bond stresses increased most rapidly with slip at the loaded end of the bar; and (3) the divergence of values of bond stress for a given slip at the two ends of a bar increased with length of embedment.

William N. Findley, USA

1340. **S. Kadziałko, "Concerning tunnel design (Unwaga nad obliczaniem tuneli),"** *Inżyn. Budown.*, Dec. 1947, vol. 4, pp. 485–491.

The design of "natural arches" for tunnel constructions is discussed, starting with remarks about previous design layouts (Ritter, Kommerell, Engesser, Protodyakonov, Suquet, Terzaghi). The strength of such arches covering natural or artificial subterranean cavities is discussed on the basis of elementary theory.

W. Olszak, Poland

1341. **T. Kluz, "Airplane hangars (Hangary lotnicze),"** *Inżyn. Budown.*, vol. 4, Sept.-Oct., 1947, pp. 359–364; vol. 4, Dec. 1947, pp. 491–495; vol. 5, Mar. 1948, pp. 108–115; vol. 5, Apr. 1948, pp. 169–171.

Practical and theoretical principles of hangar construction are discussed. After disposing of the general requirements, in particular for hangars of large span, the advantages and disadvantages of various designs are taken up. Hangars of steel, reinforced concrete, and prestressed concrete are given particular attention.

Cost and weight data are given, with shell-type construction emphasized. Prefabrication is mentioned and the TK (Tomasz Kluz) system is shown to result in material saving. Finally, various designs of hangar doors are discussed.

W. Olszak, Poland

1342. **E. Freyssinet, "Structures of prestressed concrete erected to contain or retain liquids (Ouvrages en béton précontraint destinés à contenir ou à retenir des liquides)"** (with summaries in English and German), *Publ. int. Ass. Bridge Struct. Engng.*, Prelim. Publ. 3rd Congr., 1948, pp. 343–360.

Some of the most revolutionary techniques in prestressed reinforced concrete construction are reviewed in this paper. The author constructs pipes of concrete having 17,000 psi compressive strength, and makes them by a continuous process using heat, vibration, and pressure, and a very elaborate mold. The bars are prestressed longitudinally and transversely. Such pipes of 3, 4, and 5-in. diam are in use in Algeria and are able to stand pressures of 80 to 250 psi, and remain completely watertight. The author describes briefly the theoretical background of this new type of construction and also gives some information on methods used to prestress water and wine reservoirs, and large dams. The paper is very general in character and gives few details. It shows, however, extraordinary possibilities of development in reinforced concrete.

A. J. Durelli, USA

1343. **C. Krzywicki, "Five examples of frame computation based on theory of plasticity"** (in Polish), *Inżyn. Budown.*, Dec. 1947, vol. 4, pp. 495–498; Mar. 1948, vol. 5, pp. 120–122.

Five examples of one, two, and multibay frames are used to illustrate their computation and dimensioning on the basis of the design condition that the yield point is reached at the most highly stressed cross section. The equations for these frames are derived, and their application to various types of load is illustrated by numerical examples.

W. Olszak, Poland

1344. **Jan Szanser, "Three-moment equation method"** (in Polish), *Inżyn. Budown.*, June 1947, vol. 4, pp. 274–279.

The three-moment equation can be used for studying continuous beams with constant or variable moment of inertia by equalizing the angles of end rotation. Tables for use with beams of variable cross section are presented.

W. Olszak, Poland

1345. **A. Paduart, "Large span bridges of prestressed concrete in Belgium (Ponts de grande portée en béton précontraint réalises en Belgique)"** (with summaries in English and German, *Publ. int. Ass. Bridge Struct. Engng.*, Prelim. Publ. 3rd Congr., 1948, pp. 325–332.

Reinforced prestressed concrete techniques permit light and efficient structures. However, some questions concerning its behavior limit its wide use. The author describes an experimental beam 90 ft long and 2.7 ft high, with an arch of 1 ft, which was built to check deformations and ease of prestressing. Straight tie rods $1^5/_8$ in. in diameter located in the open side of the inverted U section, were used for prestressing. The concrete work was finished in a day. Static and dynamic tests are compared with theoretical calculations. A pedestrian bridge 140 ft long was built on the basis of this experience.

A. J. Durelli, USA

1346. **Georg Wästlund and Per Olov Jonson, "Investigation of formation of cracks in reinforced concrete structures (Étude sur la fissuration des ouvrages en béton armé)"** (with summaries in English and German), *Publ. Int. Ass. Bridge Struct. Engng.*, Prelim. Publ. 3rd Congr., 1948, pp. 215–225.

This paper deals with the influence on crack formation of the following factors: diameter of bars, ratio of reinforcement, quality of concrete, and surface properties of reinforcement. Systematic laboratory tests were made and observations taken on seven existing bridges. The authors found that the widths of cracks increase almost linearly with the diameter of the reinforcing bars. They also found that for negative moment the width of cracks is reduced when the bars are distributed across the whole flange of the beam.

A. J. Durelli, USA

1347. **J. R. Robinson, "The compensation of arches (La compensation des arcs),"** *Ann. Ponts Chauss.*, Jan.-Feb. 1948, vol. 118, pp. 11–39.

This paper deals with the analysis of compensating or correc-

tive forces which may be introduced in an arch in order to reduce the maximum stresses, or to obtain a better positioning of the line of pressure. Several means of achieving compensation are considered, such as the application of jacks at one section of the arch, the use of temporary hinges which can be blocked at a predetermined stage, and the definite displacement of a hinge.

Choice of the method of compensation and the procedure for determining the optimum compensation are discussed. General formulas are given for the calculation of the compensating forces. From these, special expressions are derived for fixed arches as well as for one, two, and three-hinged arches. Several methods of compensation are considered in each case. No examples to illustrate the application of the formulas are given.

Dana Young, USA

Rheology (Plastic, Visco-Plastic Flow)

(See also Revs. 1315, 1318, 1329, 1343, 1360, 1376, 1425)

1348. F. Levi, "On static effects of viscous phenomena (Sugli effetti statici dei fenomeni viscosi)," *R. C. Accad. Lincei*, Mar. 1948, ser. 8, vol. 4, sem. 1, pp. 306–311.

In an earlier paper [*R. C. Accad. Lincei*, 1948, ser. 8, vol. 4, sem. 1, Jan., pp. 65–70; Feb., pp. 197–201] the author deduced the law of variation with time of the state of tension in a body similar to a St. Venant solid when it is subjected to a plane complex stress system such as a Volterra distortion, and when the effect of Poisson's ratio is neglected. The present paper extends those results. The author discusses qualitatively the effects on structures of permanent loads, distortions due to contraction, temperature changes, settling of supports, eventual plastic deformation, and artificially impressed deformations.

Edward Saibel, USA

1349. N. K. Snitko, "On the theory of strength of materials with regard to their structure" (in Russian), *J. tech. Phys. (Zh. tekh. Fiz.)*, June 1948, vol. 18, pp. 857–862.

Plastic yielding of a polycrystalline metal under triaxial stress with a given ratio of the principal stresses is assumed to occur at principal stress values that are the arithmetic means of the corresponding values for single crystals of all orientations for the same ratio of the principal stresses. The interaction between the grains is neglected, and the results deviate up to 5 per cent from the Huber-Mises yield criterion. No reference is made to G. I. Taylor's method of calculating approximately the interaction between the grains.

E. Orowan, England

1350. W. T. Lankford and E. Saibel, "Some problems in unstable plastic flow under biaxial tension," *Trans. Amer. Inst. min. metall. Engrs.*, 1947, vol. 171, pp. 562–573.

In this paper necking criteria are derived for the following cases of biaxial loading: (1) A flat plate on which the loads are directly applied; (2) a thin-walled tube on which the tangential load is applied by hydraulic pressure in the tube, and the axial load by hydraulic pressure in the tube plus directly applied load on the tube; (3) a sheet, clamped over a circular opening and loaded by uniformly applied hydraulic pressure (bulging).

The authors assume that necking will occur when either the tensile load or the hydraulic pressure reaches a maximum. According to their derivation the uniform strain at which necking occurs would not merely be determined by the state of stress, but very strongly by the method of loading as well. So they conclude, for instance, that in the case of balanced biaxial tension the uniform strain at necking in the tube test is less than half the uniform strain at necking in the bulge test, while in the case of direct loading the strain apparently would be intermediate.

They also conclude that in the case of direct biaxial loading the uniform strain at necking would be independent of the ratio of the principal stresses, thus being equal to the uniform strain at necking in simple tension.

J. H. Palm, Holland

1351. F. E. Piech and W. E. Gloor, "The use of the Rossi-Peakes flow tester in measuring the apparent viscosity of plastics at temperatures from 115 to 175 C," *Amer. Soc. Test. Mat. Bull.*, no. 151, Mar. 1948, pp. 70–75.

Experiments in the measurement of the viscosity of plasticized cellulose acetate and ethyl cellulose by means of flow through a capillary in the range of their flow temperatures are described. Discrepancies between observed behavior and theory for slow flow of a viscous liquid are discussed. Considerable difference was observed in the extent to which the two materials departed from the behavior prescribed by the elementary theory. Interchange of figure captions 3 and 4 increases the difficulty of reading the paper.

William N. Findley, USA

1352. V. V. Sokolovski, "On a form of representation of stress components in the theory of plasticity" (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, July 11, 1948, vol. 61, pp. 223–225.

The paper presents a new form of expressions for the six rectangular components of stress at a point.

Alexander Hrennikoff, Canada

(EDITOR'S NOTE: A six-page translation of this paper has been prepared by Dr. Hrennikoff; the editor can supply photostats at the usual rate.)

1353. L. A. Galin, "Plane elastic-plastic problem. Plastic regions around circular holes in plates and beams," *Brown Univ. Rep.*, no. All-T1/34, Dec. 1947, pp. 1–34 (transl. from *Appl. Math. Mech., Prikl. Mat. Mekh.*, 1946, vol. 10, pp. 365–386).

The paper is concerned with the stresses in a finite elastic-plastic slab with a circular hole when the state of stress at infinity and the (uniform) pressure acting on the boundary of the hole are given. The line separating the plastic from the elastic region is determined approximately by function-theoretical methods under the tacit assumption that this line includes the hole. This assumption is not necessarily justified.

Courtesy of Mathematical Reviews W. Prager, USA

1354. R. Hill, "A variational principle of maximum plastic work in classical plasticity," *Quart. J. Mech. appl. Math.*, Mar. 1948, vol. 1, pp. 18–28.

The author proves the following principle of maximum plastic work for a St. Venant-Levy-Mises material assuming the Mises yield condition to be satisfied everywhere throughout a finite mass of this material, no element of which is being unloaded. If u_i are the flow velocities on the bounding surface S of the mass, n_j is the unit outward normal to S, σ_{ij} is the actual system of stresses which occur, and $\bar{\sigma}_{ij}$ is any other system satisfying the equilibrium and yield conditions, then

$$\int_S \bar{\sigma}_{ij} n_j u_i \, dS \geq \int_S \sigma_{ij} n_j u_i \, dS$$

where equality holds if, and only if, the systems of stress differ by a hydrostatic state of stress alone.

It is pointed out that, in general, the boundary conditions cannot be prescribed arbitrarily if the yield condition is to be everywhere fulfilled throughout the region considered. In this connection the following uniqueness theorems are proved: (1) "If

arbitrary velocities are taken over the surface, then there is either no possible corresponding stress distribution for which the whole mass is plastic, or, if one exists, it is unique (apart from a uniform hydrostatic pressure)." (2) "...if the stresses are prescribed over the surface, then either no completely plastic state of stress exists, or if one exists, it is unique." The method shows that both the maximum work principle and the Castigliano (minimum) principle of elasticity are special cases of a more general theorem.

The maximum principle is applied to the problem of combined torsion, torsion and bending of a prismatic bar with constant cross section. The relation between the present principle and the heuristic principle of maximum plastic effort proposed by Sadowsky is discussed. It is noted that only in those cases where maximizing the work is identical with maximizing the effort (that is, load) will the Sadowsky principle furnish the correct answer.

H. L. Greenberg, USA

1355. **S. Basu, "Elasto-viscous effect in shellac"** (in English), *Indian J. Phys.*, Feb. 1948, vol. 22, pp. 55–63.

Elasto-viscous properties of T. N. shellac as well as of partially polymerized samples have been measured. The total extension has been found to be due to three distinct processes: true elastic deformation, delayed elastic deformation, and a viscous flow. Explanation of these various processes has been given by attributing an isocolloidal structure to shellac. Calculation of the relaxation time has shown that the dispersed phase consists of micelles formed due to secondary forces between the molecules. The dispersion medium consists of monometric particles.

On partial polymerization, the polymeric products have been found to form a part of the dispersion medium, increasing the viscosity of it. The low value of the extension has been explained by the granular nature of the micelle. R. C. Binder, USA

Failure, Mechanics of Solid State

(See also Revs. 1321, 1425)

1356. **W. M. Wilson and J. L. Burke, "Rate of propagation of fatigue cracks in 12 × 3/4-in. steel plates with severe geometrical stress-raisers,"** *Weld. Res. Suppl.*, Aug. 1948, vol. 13, pp. 405–408.

Charpy tests and fatigue tests of normalized killed steel, rimmed steel as-rolled, and strain-aged rimmed steel made at stress levels of ±16,000 psi showed that: (a) A jeweler's saw cut 0.011 to 0.012 in. wide produced as rapid a rate of crack propagation as a fatigue crack; (b) temperature variation from −40 to 120 F had no significant effect on rate of fatigue crack propagation; (c) no correlation exists between results of notched-bar Charpy tests and rate of propagation of fatigue cracks.

Hans F. Winterkorn, USA

1357. **E. G. Thomsen, I. Lotze, and J. E. Dorn, "Fracture strength of 75S-T aluminum alloy under combined stress,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1551, July 1948, pp. 1–30.

The authors report on an experimental investigation of the effect of combined stresses on the fracture strength of 75S-T aluminum alloy. They used the standard practice of applying axial load and internal pressure to tubular specimens until fracture was produced. Their test specimens were machined from aluminum-alloy drawn tubes to 3.9-in. outside diameter, 0.125-in. wall thickness, with a cylindrical gage length of 18 in.

They present their test results graphically by plotting the true stress at fracture in the axial direction as ordinate against the true stress in the circumferential direction as abscissa. The test points align along a line representing the maximum shearing stress theory, so that in their conclusions the authors state that "under biaxial stresses substantial agreement was found with the critical maximum shear stress law for fracture."

Some results are reported on the influence of mechanical fibering due to the manufacturing processes by which the aluminum-alloy tube was produced and which resulted in lower transverse than the longitudinal fracture stresses. The authors state furthermore that increasing the minimum hydrostatic tension appears to decrease the critical shear stress for fracture when the fracture surface cuts the lines of mechanical fibering, and that the critical shear stress at fracture was practically independent of the effective strain preceding the rupture over the range which they had studied.

It is regrettable that in a report like the present one symbols are used which do not comply with well-established standards. The authors use symbols such as τ_{zz} for denoting normal stresses. Such a practice should be discouraged and some uniformity should be used in reporting investigations of this kind which have been carried out at so many institutions during recent years.

A. Nadai, USA

1358. **A. Boodberg, H. E. Davis, E. R. Parker, and G. E. Troxell, "Causes of cleavage fracture in ship plate—tests of wide notched plates."** *Weld. Res. Suppl.*, Apr. 1948, vol. 13, pp. 186–199.

Results of tests of 3/4-in. steel plates from 3 to 108 in. wide containing the severe stress raiser used in the tests of Review 1361 are reported for the following special steels: semikilled (as-rolled and normalized), fully killed and alloy (as-rolled), and fully killed (quenched and drawn). Tests of plates of different width were conducted at temperatures from −78 to +152 F. The results show a marked transition from ductile to brittle-type failures at temperatures of about −50 F for the alloy steel, +70 F for one of the semikilled steels and +20 F for the other steels tested.

Energy absorbed to maximum load and percentage of shear fracture were used to evaluate the transition. It was observed that the average strength of notched plates of 3/4-in. thickness decreased markedly as the width was increased from 3 to 24 in. but that further increase in width made little difference.

Results of additional tests indicated that the transition temperature was influenced by specimen thickness and amount of reduction occurring during rolling. Crack propagation velocities were also measured. Velocities from 200 to 6600 fps were measured in wide plates having cleavage fracture. High velocities were found in the high-stress region near the notch, and low velocities in low-stress regions. William N. Findley, USA

1359. **E. Saibel, "A thermodynamic theory of the fracture of metals,"** *Trans. Amer. Inst. min. metall. Engrs.*, 1947, vol. 171, pp. 639–664.

A theory of fracture is developed using a simple analysis based on the Eyring model of the liquid state. Assuming a liquid to be equivalent to a solid containing a distribution of holes moving through it, and the abolition of cohesion to require the formation of such holes, an energy relationship is developed as a criterion of fracture by equating the strain energy required to bring the material to fracture stress to a proportion of the latent heat of melting. The criterion is applied to completely brittle fractures where the strain energy is elastic, and to fracture following plastic flow, and satisfactory agreement with experiment is demonstrated for a wide range of published tests. However, the reviewer agrees with a point mentioned in the discussion of the paper, that there is a numerical error in the energy units used which would nullify this agreement with experiment.

The theory assumes all the strain energy to be available for

causing fracture, and so determines a total energy criterion dependent only on the latent heat and change of volume on melting apart from an arbitrary term to allow for initial energy density. As pointed out in the discussion, this assumption violates the known fact that a large proportion of the energy of plastic flow is transformed into heat, and it does not provide an explanation of the marked influence of heat treatment on the energy required for fracture. E. H. Lee, USA

1360. L. D. Hall and E. R. Parker, "Effect of residual tension stress on the fatigue strength of mild steel," *Weld. Res. Suppl.*, Aug. 1948, vol. 13, pp. 421–425.

Two types of experiments were performed to determine the effects of residual stresses upon the endurance limit of mild steel specimens. The first type of experiment used test specimens with longitudinal weld beads along their center line. A part of these specimens were "stress relieved" by subjecting them to a one per cent strain. There was no appreciable difference in the endurance limit of the strained and unstrained welded specimens. From this it was concluded that under certain conditions residual stresses could exist in welded structures without decreasing the fatigue strength of the material. A second type of experiment was conducted on notched specimens with residual tensile stress, no residual stress, and residual compressive stress, at the base of the notch. The fatigue strengths of the specimens were decreased by residual tensile stress and increased by residual compressive stress. Irwin Vigness, USA

1361. W. M. Wilson, R. A. Hechtman, and W. H. Bruckner, "Cleavage fracture of ship plates as influenced by size effect," *Weld. Res. Suppl.*, Apr. 1948, vol. 13, pp. 200–207.

Results of tests of $^3/_4$-in. ship plates from 12 to 72 in. wide containing the severe stress raiser used in the tests of Review 1358 are reported for the following materials: rimmed steel and killed steels both as-rolled and normalized. Tests of different width plates were conducted at temperatures from −73 to +160 F. The results show a marked transition from ductile to brittle-type failures at temperatures of about 30 F for killed steel and 110 F for rimmed steel. Total energy absorbed to fracture and percentage of shear fracture were used to evaluate the transition. The total energy absorbed at temperatures above the transition was many times greater than below the transition. All steels had low energy absorption at temperatures below zero.

It was observed that the average strength decreased with an increase in width of plate but increased with percentage of shear fracture. Both elastic and plastic strain were much greater at the stress raiser than elsewhere, up to the start of fracture. Tests of wide plates with punched holes and sheared edges showed cleavage fractures, some reduction in strength and great reduction in energy absorption. William N. Findley, USA

Material Test Techniques

(See also Revs. 1321, 1351, 1365, 1426)

1362. A. W. Nolle, "Methods for measuring dynamic mechanical properties of rubber-like materials," *J. appl. Phys.*, Aug. 1948, vol. 19, pp. 753–774.

Five methods are described for measuring the dynamic modulus of rubber-like materials under small sinusoidal strain variations at frequencies from 10^{-1} to 10^5 cps and at temperatures from −60 to +100 C.

The methods are: (1) Measurement of the frequency and decrement of a pendulum in which the restoring force is supplied by the rubber specimen (0.1–25 cps); (2) measurement of the resonant frequency and band width (defined as the difference between the frequencies for which the amplitude is $1/\sqrt{2}$ times the maximum amplitude) of a rubber reed oscillated by a phonograph cutting head (10–500 cps); (3) measurement of the acoustic phase velocity and attentuation of longitudinal waves in a thin rubber strip in the absence of standing waves (1–40 kc per sec); (4) measurement of the band width and resonant frequency of a longitudinal standing wave of half wave length in a thin rubber strip (below 300 cps); and (5) measurement of the change in resonant frequency and change in band width for longitudinal vibration of a metal rod excited by magnetostriction when a rubber sample is pressed against the end of the metal rod (12–120 kc per sec).

Apparatus used for each method is described, the derivation of formulas for calculating the dynamic modulus are included, and the advantages and limitations of the five methods are discussed. Results of tests using these methods are to be published in the *Journal of Polymer Science*. The author differentiates between the dynamic elastic modulus and a complex modulus which includes the effect of viscosity. William N. Findlay, USA

1363. C. Sălceanu and M. Borneas, "Experiment concerning elastic homogeneity of metals (Expériences concernant l'homogénéité élastique des métaux)," *C. R. Acad. Sci. Paris*, May 3, 1948, vol. 226, pp. 1422–1424.

The authors performed experiments with thin wires made of steel, iron, silver, aluminum, and copper. The wires formed the suspension of a torsional pendulum and were tested under different tensile stresses. With increasing tensile stress a decrease in natural period depending upon the type of material could be observed, reaching a limit which the authors interpret as corresponding to a state of elastic homogeneity.

B. K. Bernhard, USA

1364. Staff of Forest Products Laboratory, "Methods for conducting mechanical tests of sandwich constructions at normal temperature," *For. Prod. Lab. Rep.*, no. 1556, Mar. 1948, pp. 1–26.

The methods described in this report cover procedures for testing sandwich constructions in compression, tension, bending, and shear at normal temperatures. They are based on techniques developed at the Forest Products Laboratory in connection with the research program of the Army-Navy-Civil Committee on Aircraft Design Criteria.

Detail drawings of the apparatus used in the various tests are given, and also the details of the procedures. It is considered that these methods are preliminary and may be revised as additional data become available. Marshall Holt, USA

Mechanical Properties of Specific Materials

(See also Revs. 1339, 1346, 1355, 1356, 1357, 1358, 1360, 1361)

1365. G. Sachs, L. J. Ebert, and W. F. Brown, Jr., "Comparison of various structural alloy steels by means of the static notch-bar tensile test," *Trans. Amer. Inst. min. metall. Engrs.*, 1947, vol. 171, pp. 605–621.

To differentiate between steels having identical properties in the conventional tensile test, but different service performances, the authors have statically tested bars provided with a sharp 60-deg circumferential V-notch, and studied three types of SAE oil-quenched and tempered steels, containing 3.5 per cent Ni, 1 per cent Cr, and 2 per cent Mn, respectively. On the last of them the influence of two different heats has also been investigated. The regular tensile properties, the concentric and eccentric notch strength, and the notch ductility (that is, the contraction in area

at the root of the notch) of these steels, are given as a function of the tempering temperature. The diagrams are also replotted with the ordinary tensile strength (strength level) as abcissa.

It is found that at strength levels exceeding 175,000 psi, the concentric notch strength reaches a maximum and then decreases rapidly. The eccentric notch strength values show approximately the same trend above 200,000 psi. Furthermore, in this range and for the same strength level, there are large differences tween the notch strengths of the different steels (and even between different heats of the same steel). The 3.5 Ni steel shows considerably higher notch properties than the two others. Finally the ratio of notch strength to tensile strength is found to be a definite function of the notch ductility. This function, of course, is different for the concentric and eccentric notch tests.

Ch. Massonnet, Belgium

1366. **Joseph Marin, "Static and dynamic creep properties of laminated plastics for various types of stress,"** *Nat. adv. Comm. Aero. tech. Note.* no. 1105, Feb. 1947, pp. 1–67.

Data from the following tests of five laminated plastics are reported: Creep tests in tension, bending, and torsion; creep tests in fluctuating tension; static tension, compression, bending, and torsion tests; and tests to determine the effect of repeated overstressing on the static strength properties. The ultimate strength was only slightly affected by repeated overstressing. No analysis or comparison of the results of the different types of creep tests is given.

William N. Findley, USA

1367. **C. T. Evans, Jr., "Materials for power gas turbines,"** *Trans. Amer. Soc. mech. Engrs.*, Aug. 1947, vol. 69, pp. 601–608.

The steels selected for the construction of three different types of power gas turbines are described, together with reasons for their choice and some of the metallurgy involved. Design features of the turbines are also described.

Data are presented on the time-for-rupture at constant stress found in tests of 19-9 W-Mo alloy, 19-9 DL alloy, and wrought S-590 alloy at 1200 F and at several stresses. Data for tests of S-590 alloy at 1800 F are also presented.

William N. Findley, USA

1368. **J. S. Offutt and C. M. Lambe, "Plasters and gypsum cements for the ceramic industry,"** *Bull. Amer. ceram. Soc.*, Feb. 1947, vol. 26, pp. 29–36.

Physical properties of a new plaster, alpha-gypsum, suitable for use in molds in the ceramic industry are described and its advantages enumerated.

A. R. Bobrowsky, USA

1369. **S. B. Newman and J. H. Curtiss, "A statistical analysis of some mechanical properties of manila rope,"** *J. Res. nat. Bur. Stands.*, Dec. 1947, vol. 39, pp. 551–559.

This is an interesting contribution to knowledge of the relationships between (1) circumference, weight, and strength, and (2) nominal diameter of 3-strand manila ropes. It sums up the results of tests of 863 samples, ranging in size from $^3/_{16}$ in. to 3 in. The results are analyzed by modern statistical methods.

The mean values of the dependent variable Y (circumference, weight, or strength) were expressed as functions of the nominal diameter X by an equation of the form $\log Y = a + b \log X$ where the parameters a and b are determined by the standard unweighted least square method. In one case b was arbitrarily taken as unity. The standard errors of a and b were very small.

The above equations were next transformed into the form: $Y = 10^{a+1.1562}X^b$, the corresponding data being summarized by curves and tables. The following general statements can be deduced from the coefficients and exponents found for the latter equations: (1) In the case of the relations between circumference and diameter, the coefficient of D is significantly larger than three, the value suggested by the definition of nominal diameter; (2) in the weight-diameter and strength-diameter relationships, the exponent of D is significantly less than the value of two suggested by the simplest physical theory; (3) in the relation between strength and weight W, the exponent of W is significantly different from unity.

The mean strengths obtained by the authors for sizes of 1-in. diam and more are greater than those observed by Stang and Strickenberg in 1921.

D. DeMeulemeester, Belgium

1370. **R. L. Moore, "Bearing strengths of some aluminum-alloy sand castings,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1523, Aug. 1948, pp. 1–19.

Tests were made to evaluate the bearing strength of aluminum-alloy sand castings and their relation to tensile properties. Comparisons between individually cast bars and specimens machined from slabs emphasized the effect of size and form of casting upon mechanical properties. Bearing-strength characteristics of $^1/_8$ and $^1/_4$-in. thick specimens with respect to edge distance and ratios of specimen width to pin diameter were similar to those for common wrought-aluminum alloys. The ratios of bearing yield to tensile yield strengths for castings were about 20 per cent higher than for wrought materials; the ratios of bearing-ultimate to tensile-ultimate strengths were about the same.

T. J. Dolan, USA

1371. **J. A. Loe and F. N. Sparkes, "The deterioration of concrete: Some factors affecting the resistance of concrete to frost action (Destruction du béton: Effet de l'influence néfaste du gel)"** (with summaries in English and German), *Publ. int. Ass. Bridge Struc. Engng.*, Prelim. Publ. 3rd Congr., 1948, pp. 201–214.

The authors study systematically the influence of water-cement ratio, cement content, aggregate grading, type of aggregate (chalk and limestone), admixtures (Vinsol resin and aluminum powder), and number of cycles of freezing and thawing on the crushing strength of concrete. Tests were run for specimens of ages from seven days up to 110 days. The authors consider that the technique they used in the laboratory subjected the specimens to much more severe conditions than those prevailing in normal weather. The most important factor influencing the resistance to frost action is water-cement ratio. The authors give recommendations for the design of concrete mixtures.

A. J. Durelli, USA

1372. **B. W. Mott and J. Thompson, "Investigations on aluminum alloys of high strength at room temperature,"** *Metal Treatm.*, 1947, vol. 14, winter, pp. 227–238; 1948, vol. 15, spring, pp. 33–46; summer, pp. 91–105.

An extensive survey of the tensile properties, hardness, resistance to stress corrosion and fatigue, and microstructure of wrought aluminum alloys containing various combinations and amounts of magnesium, zinc, and manganese as the principal alloying elements is given.

J. D. Lubahn, USA

Mechanics of Forming and Cutting

(See also Rev. 1350)

Hydraulics; Cavitation; Transport

(See also Revs. 1393, 1430)

1373. J. Williamson, "Drag velocity in relation to production of turbulent energy and loss of head in pipes in the V^2-law region of flow," *J. Instn. civ. Engrs.*, May 1948, vol. 30, pp. 251–276.

The correlation between Manning's roughness factor n and Nikuradze's roughness size K is satisfactory in the V^2-law range of friction flow in pipes up to values of relative roughness $(D/K) = 1000$ (D = diameter). At higher values, the two laws diverge rapidly, experience indicating that Manning's formula is valid up to $(D/K) = 4000$. It is reasoned that the mass rate of eddy production near the boundaries is independent of the roughness size, and is proportional to a "drag velocity" V_D at a distance $1.5K$ from the pipe wall. This distance is derived from velocity distributions for uniform pipe flow. The temporal mean drag velocity V_D is close to $1.15V_q(K/D)^{1/8}$ wherein V_q is the average velocity.

The method is then used to estimate local rates of eddy production, and to explain the energy losses in nonuniform flow occurring for transition lengths of plates and pipes, for bends and for converging sections. The development of spiraling flow in pipes and open channels is also explained. Arthur T. Ippen, USA

1374. W. Hensen, "The influence of an ice cover over a river on the water movement and the roughness of the bed (Der Einfluss einer Eisdecke auf den Abfluss und die Bettrauhigkeit in einen Fluss)." *Bautechnik*, June 1948, vol. 25, pp. 133–138.

The values of the roughness coefficient n in Forchheimer's formula $v = (R^{0.7}J^{0.5})/n$ (v = mean velocity of the current; R = hydraulic radius of the gaging cross section; J = surface slope) have been calculated on the basis of two discharge measurements carried out on the Elbe river, completely covered over by a 2-ft layer of ice. Values calculated are n_1 for the whole stream, n_2 for the ice part of the wetted perimeter, and n_3 for the bed of the stream. Attempts have been made to separate the resistance due to roughness, and defined by the values n_2 and n_3, from the resistance ascribed to the variations of the cross section along the axis of the stream, by assuming $n_1 = {}^1/_2(n_2 + n_3) + n_u$, where n_u is called the coarse roughness. The values obtained from the two measurements for n_2, n_3, and n_u do not seem much in accord, but they ought to prove that n_u has always an outweighing influence.

The layer of ice resulting from the grouping of wandering ice blocks appears very rough when recently formed, but grows smoother owing to the action of the current.

Giulio de Marchi, Italy

Incompressible Flow: Laminar; Viscous

(See also Revs. 1351, 1395, 1400, 1401, 1405, 1431, 1434, 1437)

1375. N. Tetervin, "Laminar flow of a slightly viscous incompressible fluid that issues from a slit and passes over a flat plate," *Nat. adv. Comm. Aero. tech. Note*, no. 1644, June 1948, pp. 1–40.

In a very large container of still fluid, the flow which enters from a slit at the intersection of two of the walls and passes over one of them is investigated in a region far enough from the slit for the boundary-layer equations to be valid. The velocity distributions in planes normal to the direction of main flow are assumed similar, and the partial differential equation of the boundary layer is reduced to a third-order nonlinear ordinary differential equation by a suitable substitution of variables (essentially a generalization of the substitutions employed in the problems of the boundary layer over a flat plate and the jet). This ordinary differential equation is then integrated numerically for the required boundary conditions. The solution gives the velocities at points in the fluid and the surface friction at points on the plate.

Among some of the specific results obtained are that the width of the disturbed region increases as the $^3/_4$ power of the distance from the slit, and that the surface friction is independent of the viscosity and varies inversely as the $^5/_4$ power of the distance from the slit. J. S. Isenberg, USA

1376. H. Umstätter, "The role of Maxwell's relaxation theorem in the macromolecular organic chemistry (Über die Rolle des Maxwellschen Relaxationstheorems in der makromolekularem organischen Chemie)." *Kautschuk Gummi*, Jan. 1948, vol. 1, pp. 11–15.

The author derives the relationship between the viscosity of a macromolecular solution and the molecular weight, using Maxwell's definition of viscosity. From the correct interpretation of Maxwell's relaxation theorem, the equation of the flow curve can be derived. Combining this equation with da Andrade's viscosity equation the relaxation displacement law is obtained, from which the relation between viscosity and particle weight is then determined.

The author's line of reasoning is as follows: Newton's definition of viscosity can be applied to continuous (micromolecular) fluids only. Macromolecular fluids contain particles of finite size and therefore the law of continuity cannot be applied to them. Maxwell's definition of viscosity, as the product of the coefficient of elasticity E and the time of relaxation τ, is independent of any hypothesis and is applicable to macromolecular solutions.

Flow curves of such solutions, that is, $\ln(dv/dy)$ versus $\ln P$ curves, where v is the velocity of flow, dv/dy the velocity gradient, P the shear stress, will not be straight lines passing through the origin (as for Newtonian liquids) but will have a point of inflection at which a critical velocity is reached. At this velocity dispersion takes place. The probability that the velocity of propagation of the disturbance is between c and $c + dc$ can be expressed as $W = [Mc^2/(2\pi RT)]^{1/2}\exp(Mc^2/RT)$ where M is the mass of the energy center, R the gas constant, and T the absolute temperature.

Noting that the velocity of propagation of a disturbance is equal to $(E/\rho)^{1/2}$ where ρ is the mass density, da Andrade's equation is obtained as $d(\ln\eta)/dT = ME/(\rho RT^2)$, η being the viscosity. Combining this with Maxwell's definition $\eta = E\tau$, the relaxation displacement law is obtained in the form

$$\rho RT^2 d\eta/dT = M\eta^2/\tau$$

It shows that the time of relaxation increases with increasing particle weight and decreases with increasing temperature.

The mass M of the energy center can be determined from two flow curves taken at different temperatures and measuring τ in both cases. Since M is a function of the concentration, the weight of the particles can only be determined if the viscosity concentration function is known.

The paper is concise and clear. A list of notations would be helpful. Nicholas Sag, Australia

1377. N. L. Ghosh, "A note on the equilibrium of fluid matter in a steady differential rotation" (in English), *Bull. Calcutta math. Soc.*, Sept. 1947, vol. 39, pp. 131–138.

The paper considers the problem of the steady differential rotation of a mass of frictionless fluid about an axis of symmetry for the case when the gravitational potential is a function of the fluid density ρ alone. It is shown that the latter assumption imposes

certain restrictions on the density distribution of the fluid mass. In particular, $\nabla^2\rho$ and $|\nabla\rho|$ must both be functions of ρ alone. Assuming that the gravitational potential is produced by a very heavy central core, the gravitational effect of the fluid itself being negligible, it is shown that the fluid may remain in a configuration of equilibrium in the form of a spheroid with differential motion around the heavy core, or as an anchor ring concentric with the core. It is observed that in both examples the density increases while the angular velocity diminishes outwardly.

Robert C. F. Bartels, USA

Compressible Flow, Gas Dynamics

(See also Revs. 1397, 1399, 1403, 1423)

1378. **R. Siestrunck and J. Fabri, "Approximate determination of the corrections due to induced velocities in axial machines with supersonic relative speeds (Détermination approchée des corrections dues aux vitesses induites dans les machines axiales à vitesse relative supersonique),"** *C. R. Acad. Sci. Paris*, May 31, 1948, vol. 226, pp. 1795–1797.

The authors attempt to calculate the distribution of thrust of an airscrew in the supersonic speed range; the problem seems to be mainly of theoretical interest.

The calculation is performed by means of the strip theory, based on the fact that the influence of a single element of the airscrew is limited to the Mach cone. Two steps of the approximation are mentioned, taking into account the variable Mach numbers along the blades. The first one seems to use the dependence of wing section characteristics on the local Mach number, while the second step uses the pressure distribution over an airfoil, computed from the two-dimensional supersonic flow of the wing section of the propeller. The result of the calculation is plotted in a chart. No comparison with experimental results is given.

E. Abody-Anderlik, Hungary

1379. **K. Margolis, "Supersonic wave drag of nonlifting swept-back tapered wings with Mach lines behind the line of maximum thickness,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1672, Aug. 1948, pp. 1–34.

This paper represents a continuation of a program of calculations of wave drag at zero lift of swept-back wings at supersonic speeds. Variations in taper and aspect ratio are considered for wings having symmetrical double-wedge airfoil section, the calculations being based on the linearized equations for three-dimensional supersonic flow. In the present report the case where the line of maximum thickness is swept back behind the Mach lines is considered.

The results of earlier work with subsonic lines of maximum thickness are included in this study and results are presented on the basis of constant aspect ratio and sweepback, constant taper ratio and sweepback, constant root-bending stress and sweepback, and constant taper and aspect ratio. In each case variations in the remaining geometric parameters and in the Mach number are considered. M. J. Thompson, USA

1380. **A. Fage and R. F. Sargent, "Shock-wave and boundary-layer phenomena near a flat surface,"** *Proc. roy. Soc. Lond. Ser. A*, June 17, 1947, vol. 190, pp. 1–20.

The report contains an experimental investigation of the shock pattern obtained in passing from a supersonic to a subsonic stream in the presence of a wall with turbulent boundary layer. The flow field was studied with Pitot static pressure and surface tubes, as well as by use of the shadowgraph and schlieren technique.

Normal and oblique shock waves were considered in the experiment, and it was found that with either wave system the wave pattern in the vicinity of the wall could be maintained as a continuation of the wave pattern in the main body of the stream, or could be made to fork into two shock waves. The forked configuration was found to occur when the pressure rise across the wave above the fork was greater than a certain value.

The height of the fork, as well as the angle between the two forked waves, at first increased rapidly with an increase in wave strength above this value and afterwards much more slowly. The pressure rose steeply behind the front limb of the fork, while the back limb eased the distortion of the flow field behind it. The boundary layer thickened rapidly between the two limbs of the fork and the surface friction fell steeply to zero in the intervening space. The loss of stagnation pressure, and hence the drag, was smaller in the forked region of the flow in the main wave.

Behind the base of an unforked wave the surface friction did not fall to zero, and the rise in surface pressure and the thickening of the boundary layer were less severe. The report includes a quantitative study of the experimental data. Some inference on the behavior of the flow at higher Mach numbers than those of the experimental investigation of the report are included.

H. Julian Allen, USA

1381. **S. Paterson, "The hydrodynamic theory of detonation,"** *Research Lond.*, Feb. 1948, vol. 1, pp. 221–233.

This paper is concerned with the properties of detonation waves set up when a rigidly confined solid or liquid explosive is detonated. The first two-thirds of the paper is devoted to a literature review. The author discusses briefly the hydrodynamic theory of Chapman and Jouguet for gaseous explosives, and follows up with a review of the literature (which includes some of the author's previous work) dealing with the extension of this theory to liquid or solid explosives which yield only gaseous products. The remainder of the paper is devoted to an extension to those cases where the products of the explosion may contain non-gaseous components. This extended theory is applied to a series of commercial blasting explosives, and it is found that the predictions of theory concerning the velocity of the detonation wave are in reasonable agreement with observation.

Benjamin Epstein, USA

1382. **M. I. Gurevich, "On the thin triangular wing at supersonic speed"** (in Russian), *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, May-June 1947, vol. 11, pp. 395–396.

This is an addition to the paper of Falkovich and Haskind (see Rev. 353, APPLIED MECHANICS REVIEWS, February 1948). It is shown how the solution is to be modified when part of the wing is outside of the Mach cone. D. R. Mazkevich, USA

1383. **S. A. Hristyanovich, "Approximate integration of the equations of a supersonic gas flow"** (in Russian), *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, Mar.-Apr., 1947, vol. 11, pp. 215–222.

In the case of two-dimensional motion of a compressible fluid, the equations for the potential and stream function can be written in the form

$$\varphi_\eta = \chi\psi_\eta, \quad \varphi_\xi = -\chi\psi_\xi, \quad 2\xi = \sigma - \vartheta, \quad 2\eta = \sigma + \vartheta$$

σ being a function of the speed, and ϑ the angle which the velocity vector forms with the positive x-axis. χ is a function of σ. x, y are Cartesian co-ordinates in the physical plane.

The author approximates χ in the intervals J_K, $K = 1, 2, J_1$ $[1.06 < M < 2.2]$, J_2 $[2.2 < M < 3.5]$, by functions $s_K = C_K{}^{(i)}$

$(\sigma + C_K^{(2)})^2$, where $C_K^{(s)}$ are conveniently chosen constants and M is the Mach number. Under these simplifying assumptions he obtains an explicit representation for ψ, $\psi = (\xi + \eta)^{-1}[f_1(\xi) + f_2(\eta)]$ in terms of two arbitrary functions f_s, $s = 1, 2$.

The expressions for dx and dy are not complete differentials in the simplified case, but become so after negligible changes. The solution of the initial value problem in the ξ, η-plane in the simplified case is discussed. Stefan Bergman, USA

1384. K. P. Stanyukovich, "Some exact solutions of equations of gas dynamics for centrally symmetrical flows" (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, June 1, 1948, vol. 60, pp. 1141–1144.

Certain classes of solutions of the equations of unsteady non-isentropic one-dimensional gas flow are discussed. If u, p, ρ, are the velocity, pressure, and density, then these classes are essentially of the form $u = t^{\alpha}\xi(rt^{\beta})$, $p\rho^{-\gamma} = t^{\epsilon}\sigma(rt^{\beta})$, $\rho = t^{\delta}\eta(rt^{\beta})$; here, r and t are space and time co-ordinates; α, γ, δ, ϵ are integration constants. These solutions are said to be of interest in astrophysical and in certain shock interaction problems. However, no specific boundary-value problems are mentioned.

George Carrier, USA

1385. S. Paterson, "The reflection of a plane shock wave at a gaseous interface," *Proc. phys. Soc. Lond.*, Aug. 1948, vol. 61, pp. 119–121.

A plane shock wave falls normally on the interface between two ideal gases of the same pressure and temperature but different molecular weights and different (constant) ratios of specific heats. The transmitted wave is always a shock wave while the reflected wave for most gases is either a shock wave or rarefaction wave depending on the value of the ratio of molecular weight times specific heat for the two gases. In exceptional cases (like neon-acetylene, argon-butane vapor, argon-methyl chloride vapor) the reflected wave is a shock wave for small incident shock waves but becomes a rarefaction wave for large incident shock waves, or vice versa. Quantitative criteria and their explanation are given, based on theoretical analysis. Possibility of experimental verification with schlieren photographs is mentioned.

Andrew Vazsonyi, USA

1386. S. H. Maslen, "Method for calculation of pressure distributions on thin conical bodies of arbitrary cross section in supersonic stream," *Nat. adv. Comm. Aero. tech. Note*, no. 1659, July 1948, pp. 1–19.

The present paper deals with the pressure distribution over conical bodies of noncircular cross section in a supersonic flow. The elementary conical fields are produced by a continuous distribution of sources along a straight line through the origin, the strength of the sources being proportional to the distance from the origin of the conical field. These line sources may be oriented obliquely to the flow direction. Using the potential equation for the linearized case one can superimpose these basic conical fields. In this way some interesting examples are obtained; for instance, one can calculate with reasonable amount of work the flow over cones with elliptical cross sections or with triangular cross sections with rounded corners. Extentions of the method may be made to cases of bodies in yaw, or with angle of attack, or with both. Hideo Yoshihara, USA

1387. M. Schaefer, "Formation of envelopes of Mach waves on a tangentially struck blade," *Hdqtrs. Air Mat. Comm. Dayton Transl.*, no. A9-T-11, 1948, pp. 1–29 (transl. from *T. H. Dresden Arch.*, no. 44/10, May 1943).

A study is made of the envelope of converging straight characteristics in the neighborhood of its initial point. Parametric equations for the characteristics of the other family, of the streamlines, and of the envelope are derived, using θ, the flow direction, or s, the arc length along the second family characteristic, as the parameters. Series expansions are used for the solutions. It is shown that (1) both $d\theta/ds$ and $d\alpha/ds$ (α = Mach angle) become infinite at the initial point; (2) the curvature of the second family characteristic becomes positively infinite for free-stream Mach numbers given by $\cos\alpha > \sqrt{k+1}/2$, where k is the ratio of specific heats, negatively infinite for $\cos\alpha < \sqrt{k+1}/2$, and zero for $\cos\alpha = \sqrt{k+1}/2$; (3) the greater the jump in the streamline curvature, the nearer to the wall the envelope begins.

The envelope is later replaced by a shock front whose equations are derived in a similar fashion. It is shown that the shock-front curvature is smaller than the curvature of the envelope and therefore actually lies behind the envelope.

Lester L. Cronvich, USA

1388. M. J. Lighthill, "Supersonic flow past slender pointed bodies of revolution at yaw," *Quart. J. Mech. appl. Math.*, Mar. 1948, vol. 1, pp. 76–89.

Approximations to the lift, drag, and moment of a slender pointed body of revolution whose axis is inclined at a small angle ψ to a uniform supersonic stream are obtained. The first approximations of lift and moment coefficients are the same as those obtained by Tsien [*J. Aero. Sci.*, 1938, vol. 5, pp. 480–483] by solving the linearized equation of motion. In this paper, the exact equations of isentropic flow, improved approximations of the lift and moment coefficients are given as sums of terms of orders ψ, ψ^3, ψt^2, and $\psi t^2 \log t^{-1}$, and the drag coefficients due to yaw as a sum of terms of orders t^2, $t^2 \log t^{-1}$, and ψ^2, where t is the maximum thickness. Terms of higher order are neglected. Although the flow is assumed to be isentropic and irrotational, the author argues that the effect of change of entropy through the shock wave will only be felt in these neglected terms of higher order.

C. T. Wang, USA

1389. R. Leduc and J. Villey, "Model tests on thermopropulsive nozzles (L'expérimentation des tuyéres thermopropusives sur modéles réduits)," *C. R. Acad. Sci. Paris*, May 3, 1948, vol. 226, pp. 1426–1428.

The author first states that model tests on nozzles may seem to be a priori impossible since the necessary length of path for complete combustion cannot be reduced at will. He then proposes a method for model testing whereby experiments related both to aerodynamic shape and the distribution of heat can be performed. The proposed method consists in introducing into the nozzle a body of revolution coaxial with the nozzle and of a convenient profile. The author states that this is equivalent to introducing a *fictitious combustion*. He then explains the method of determining the profile of this body by a step-by-step method using equations derived previously [*C. R. Acad. Sci. Paris*, 1935, vol. 201, p. 1471]. Ahmed D. Kafadar, USA

1390. C. B. Cohen and J. C. Evvard, "Graphical method o obtaining theoretical lift distributions on thin wings at supersonic speeds," *Nat. adv. Comm. Aero. tech. Note*, no. 1676, Aug. 1948, pp. 1–42.

A combined graphical and numerical method, based on linearized theory, is presented for calculating the lift distribution on thin flat wings at supersonic speed. The method is essentially one of evaluating the terms in an equation which was previously derived by Evvard [*Nat. adv. Comm. Aero. tech. Note*, no.

1585, May 1948] and which expresses the pressure at a point on the wing as the sum of a line integral and an algebraic function. The method is applicable to wings with both curved and straight planform boundaries, but is restricted to those regions of the wing which are not affected by interacting external flow fields. This restriction implies that the leading edge of the wing must be at least partially supersonic.

Application of the present method to two simple cases, which can also be solved analytically, indicates that the lifting pressure at a given point can be found in five to ten minutes with an average error of less than one per cent. As examples of the application of the method to more complex wing shapes, the calculated lift distribution is presented for the forward portion of a wing of circular planform, and for the outboard portion of a straight, sweptback wing with semicircular tip. The procedure for the treatment of various possible regions of complete wings is also indicated. Walter Vincenti, USA

1391. **C. C. Lin and S. I. Rubinov, "On the flow behind curved shocks,"** *J. Math. Phys.*, July 1948, vol. 27, pp. 105–129.

In the first part of this paper the authors develop the equations which express the (first order) derivatives of the state variables (pressure, velocity, etc.) in terms of the upstream uniform flow parameters and the curvature and slope of the shock. They also present a variational principle applicable to isoenergetic flows (that is, flows proceeding from a curved shock preceded by an isentropic irrotational flow).

In the subsequent sections the following results are obtained: (1) The curvature of the shock attached to a pointed body in a supersonic flow is, in the neighborhood of the nose, proportional to the curvature of the body. The coefficient depends on the nose slope and the upstream Mach number. The equations used here have been found previously by Thomas and Crocco. (2) The curvature of the Mach line (degenerate shock) attached to the nose of a cuspidal body is proportional to the nose curvature and may be positive or negative depending on the Mach number. (3) The result of Tsien and Emmons, concerning the Mach number at which a normal shock can occur along a smooth boundary, is modified to allow for the possibility of a shock with infinite curvature. A detailed discussion of the interpretation of this result is given. (4) Finally, the local calculation of the isoenergetic flow between a blunt obstacle and a curved detached shock is indicated and certain results are quoted.

George Carrier, USA

Turbulence, Boundary Layer, etc.

(See also Revs. 1373, 1375, 1380)

1392. **G. K. Batchelor and A. A. Townsend, "Decay of isotropic turbulence in the initial period,"** *Proc. roy. Soc. Lond. Ser. A*, July 21, 1948, vol. 193, pp. 539–558.

Continuing previous experimental and theoretical work (see Rev. 323, APPLIED MECHANICS REVIEWS, February 1948) the turbulence energy decay and self-preservation properties were studied for a wide range of grid Reynolds numbers and decay times. The initial period of decay is defined as that during which $u'^{-2} \propto t$, $\lambda^2 = 10\nu t$ (u'^2—mean square fluctuation, λ—Taylor's dissipation length parameter). In the final period, which occurs after long decay times, the correlation functions are completely self-preserving and the energy decay law is $u'^{-2} \propto t^{5/2}$, $\lambda^2 = 4\nu t$. The initial period is also subdivided into three regimes according to the magnitude of the Reynolds number of turbulence $R_\lambda = u'\lambda/\nu$.

Measurement of u' and λ at various distances x behind four similar biplane grids agreed with the initial period decay relations for x/M (M—Mach number) distances up to 120 to 200. The virtual origin of turbulence x_0, found by extrapolation to zero of the U^2/u'^2 and λ^2 curves, varied somewhat with the grid Reynolds number. The slopes of the U^2/u'^2 versus x/M curves were essentially constant, and the authors postulate that the slope must be wholly determined by the grid characteristics. Based partly upon data given by von Kármán for grids of different mesh-diameter ratios, the relation in terms of the grid drag coefficient is $U^2/u'^2 = (106/C_D)(x - x_0)/M$.

The double velocity correlation function g was found to maintain its form for distances between points of the order of magnitude of λ, while outside the curves contracted laterally as x/M increased. Hence the integral scale of turbulence L increases less rapidly than λ in the initial period. At high R_λ, the shape of the correlation function agreed well with Kolomogoroff's theory of local isotropy (see Rev. 863, APPLIED MECHANICS REVIEWS, May 1948). Measurement of the triple correlation coefficient k as a function of r/λ, as well as the variation found for g, showed that the initial period is characterized by a decay law in which R_λ is constant.

In interpreting the initial period, the authors note that as L, the scale of the largest eddies, increases less rapidly with distance than λ, the scale of the small eddies, the range in eddy size diminishes with time. The larger eddies lose energy through inertial transfer to the smaller eddies which are dissipated chiefly through viscosity. Therefore, viscosity becomes in time a significant factor in the energy reduction for all eddies, and a fundamental change in the decay law results. J. M. Robertson, USA

1393. **R. Comolet, "Calculation of the boundary-layer thickness in a covergent nozzle of revolution (Calcul de l'épaisseur de la couche limite dans une tuyére convergente de révolution),"** *C. R. Acad. Sci. Paris*, June 21, 1948, vol. 226, pp. 2049–2051.

The hydrodynamic equation for a body of revolution (by C. B. Millikan) is assumed, the usual boundary-layer postulates are made, and an ordinary differential equation obtained for the flow velocity at the surface of the boundary layer. This equation is solved by a quadrature, and this solution approximated. Validity is claimed for both laminar and turbulent boundary layers, and even for slight separation far downstream. The wall influence on the boundary-layer thickness is negligible where the flow is close to single source flow. L. M. K. Boelter, USA

1394. **Chia-Chiao Lin, "Velocity and temperature distributions in turbulent jets"** (in English), *Sci. Rep. Nat. Tsing Hua Univ. Ser. A Math. Phys. Engng. Sci.*, Oct. 1947, vol. 4, nos 4, 5, 6, pp. 419–450.

The correlation theory of turbulent flow, developed by P. Y. Chou, is applied to the velocity and temperature distribution in turbulent jets, both two-dimensional and axially symmetrical.

The general differential equations of motion of the fluid are simplified first by boundary-layer approximations and expressed in the form of an equation of mean motion involving both the viscous stress and the turbulent shear stress. By introducing a stream function and a dimensionless variable $\eta = y/x^m$, the equations are reduced to ordinary differential equations. By use of the fact that the momentum per unit time across any section of the jet is constant, it is possible to obtain a relation involving the exponent m which in turn determines the shape of the boundary. Thus it is shown that the width of a laminar jet increases as $x^{2/3}$ while the boundary of a turbulent jet is straight. Similar relations for the axially symmetric case show that the boundary is straight for both laminar and turbulent flow.

In a similar manner, the corresponding equations for the temperature distribution are written and reduced to a form involving unknown double correlations—this time of temperature and velocity components. Further progress with these equations can be made only after additional information regarding the turbulent shear stress and corresponding double temperature correlations has been obtained. For this purpose Chou's equations of double velocity correlations are used. In this way mean velocity and temperature distributions are obtained which can then be compared with experimental values. The comparison of both velocity and temperature distributions are made and the corresponding numerical values of parameters are chosen. In particular it is shown that the apparent viscosity of the turbulent jet is 2.36×10^3 times the viscosity of air in laminar motion.

In a final section, the microscale of turbulence is computed and compared with Taylor's relation. Howard W. Emmons, USA

1395. H. L. Alden, "Second approximation to the laminar boundary-layer flow over a flat plate," *J. Math. Phys.*, July 1948, vol. 27, pp. 91–104.

The author calls attention to the fact that the Blasius solution for the laminar boundary layer over a flat plate is not accurate near the leading edge of the plate, and that there is a singularity at the leading edge. In order to compute a second approximation a transformation of co-ordinates is made from x and $y/\sqrt{\nu}$ to parabolic co-ordinates σ and η where $x = \sigma^2 - \eta^2\nu$ and $y = 2\sigma\eta\sqrt{\nu}$. The stream function is then expanded as a power series in $\nu/U\sigma^2$ times the factor $\sigma\sqrt{U\nu}$. The first term of this series gives the ordinary Blasius solution, and the second provides a second approximation to the exact solution.

Numerical computation gives a pressure field in the vicinity of the leading edge and perceptible departures (> 1 per cent) from the Blasius solution when Ux/ν is less than 100. However the second approximation gives an infinite integrated skin friction, and hence the results can not be valid for Ux/ν less than some small value. Beyond the boundary layer the air stream behaves as a potential flow in the presence of a source distribution varying as $1/\sqrt{x}$ along the positive x-axis. [See also G. F. Carrier and C. C. Lin, "On the nature of the boundary layer near the leading edge of a flat plate," *Q. appl. Math.*, Apr. 1948, vol. 6, pp. 63–68.] Hugh L. Dryden, USA

Aerodynamics of Flight; Wind Forces

(See also Revs. 1338, 1379, 1383, 1386, 1390, 1402, 1405)

1396. A. Gessow, "Flight investigation of effects of rotor-blade twist on helicopter performance in the high-speed and vertical autorotative descent conditions," *Nat. adv. Comm. Aero. tech. Note*, no. 1666, Aug. 1948, pp. 1–27.

Flight performance measurements were made on untwisted, plywood-covered rotor blade in the subject conditions. The results were compared with measurements on a similar helicopter rotor blade having 8 deg of linear washout. They indicate that the negative blade twist appears to increase the maximum speed of the helicopter determined by blade stall, and reduce the performance losses due to stall at a given thrust coefficient and tip speed ratio. They also indicate that negative blade twist has little effect on the performance of the helicopter in power-off descent conditions. S. W. Yuan, USA

1397. J. Stack and W. F. Lindsey, "Characteristics of low-aspect-ratio wings at supercritical Mach numbers," *Nat. adv. Comm. Aero. tech. Note*, no. 1665, Aug. 1948, pp. 1–26.

As previous work indicated that the rapid rise in drag and marked stability and control changes which occur with high-aspect-ratio wings at supercritical Mach numbers are alleviated through use of low-aspect-ratio wings, a systematic study of this question was made in the Langley 24-in. high-speed tunnel. An NACA 0012 airfoil (critical Mach number 0.72) of 2-in. chord was tested with aspect ratios of ∞ to 2 and for Mach numbers up to 0.9.

The low-aspect-ratio (2 and 3) wings were found to be free of irregularities and drastic slope changes in the lift curve and to show similar improvements in the wing-moment curve. The drag at supercritical Mach numbers was found to be much less than that for the higher-aspect-ratio wings and the Mach number for significant drag increase was 0.1 higher. As the tests were made on wings with square tips, further improvements appear possible through suitable shaping of the tips.

The effects are partially explained by noting that for low-aspect-ratio wings induced characteristics are large compared with the section characteristics, and the former are determined principally by the lift coefficient and are essentially independent of Mach number. J. M. Robertson, USA

1398. L. K. Loftin, Jr., and K. S. Cohen, "Aerodynamic characteristics of a number of modified NACA four-digit-series airfoil sections," *Nat. adv. Comm. Aero. tech. Note*, no. 1591, June 1948, pp. 1–37.

Theoretical pressure distributions and experimental aerodynamic characteristics at low speeds are given for five NACA four-digit-series airfoils modified for high-speed applications. Also included are three conventional NACA four-digit-series sections. The data are provided at Reynolds numbers of three, six, and nine million and are supplementary to the extensive results in "Summary of airfoil data," *Nat. adv. Comm. Aero. adv. conf. Rep.*, no. L5CO5, 1945 by Ira H. Abbott, Albert E. von Doenhoff, and Louis S. Stivers, Jr. Comparisons are made with data for NACA low-drag sections. Max A. Heaslet, USA

1399. F. H. Hanson, Jr., and E. A. Mossman, "Effect of pressure recovery on the performance of a jet-propelled airplane," *Nat. adv. Comm. Aero. tech. Note*, no. 1695, Sept. 1948, pp. 1–35.

The title refers to the pressure at the air intake of a jet-propelled ariplane. This pressure supplements the compression otherwise developed in the turbojet engine. The purpose of the report is to assess the importance of the amount of compression achieved at the intake in terms of the performance of the airplane. With the aid of a number of calculated examples, which are presented in graphical form, the authors conclude that it is important to achieve the highest possible compression at the intake.

(The terms "pressure recovery" and "ram pressure" used in the report are the pressures developed in flight by accelerating the air up to the speed of the airplane in a continuous process. Actually, of course, greater pressures can be "recovered" by a process such as that used in the ordinary hydraulic ram.) Robert T. Jones, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 1367, 1378, 1395, 1405, 1419)

1400. F. E. Marble, "The flow of a perfect fluid through an axial turbomachine with prescribed blade loading," *J. aero. Sci.*, Aug. 1948, vol. 15, pp. 473–485.

The theory of three-dimensional flow through an axial turbomachine, associated with the variation of circulation along the

blade length, is described as an extension of the classical theory of finite wings and is simplified to a problem in axially symmetric rotational fluid motion by considering an infinite number of blades in each row. The problem is linearized by considering the vorticity generated by the blades to be transported by the mean velocity.

The linearized problem leads to well-known partial differential equations, and is solved for the radial, tangential, and axial velocity components induced by a single row of stationary or rotating blades with finite chord and prescribed loading. The particular case for which the blade chord approaches zero and the tangential velocity changes discontinuously, is associated with the theory of the Prandtl lifting line for finite wings. Because the problem has been linearized, the solution for the multistage turbomachine follows by superposition of appropriate single rows.

Analytical expressions and graphical values for the velocity components are given for a single stationary or rotating blade row of given loading with a hub-to-tip ratio of 0.6 and a blade aspect ratio of 2. R. C. Binder, USA

1401. **R. Siestrunck, "On the analysis of coaxial contrarotating propellers and their condition of optimum circulation (Sur le calcul des hélices coaxiales contrarotatives et leurs conditions d'optimum tourbillonnaire),"** *C. R. Acad. Sci. Paris*, Apr. 19, 1948, vol. 226, pp. 1246–1248.

The note indicates very briefly a procedure for treating the aerodynamic problem of a system of two coxial contrarotating propellers. A method of successive approximations is proposed, having as its starting point the potential functions of each propeller acting alone. The condition for the optimum performance of the coupled system in the sense of the minimum induced energy loss is also considered. Robert C. F. Bartels, USA

Flow and Flight Test Techniques

(See also Rev. 1351)

1402. **H. J. Goett, R. P. Jackson, and S. E. Belsley, "Wind-tunnel procedure for determination of critical stability and control characteristics of airplanes,"** *Nat. adv. Comm. Aero. Rep.*, no. 781, 1944 (issued in 1948), pp. 1–16.

This report contains a tabulation of the wind-tunnel tests needed to establish the critical stability and control characteristics of conventional airplanes. Also tabulated are illustrative data and methods of computation required to determine flying qualities from tunnel data. A similar paper [Kayten and Gerald, "Analysis of wind-tunnel stability and control tests in terms of flying qualities of full-scale airplanes," *Nat. adv. Comm. Aero. Rep.*, no. 825, 1945] covering essentially the same ground, and originally published at about the same time, contains a more detailed discussion of the fundamental problems involved in determining flying qualities from wind-tunnel tests.

John V. Becker, USA

1403. **J. Winckler, "The Mach interferometer applied to studying an axially symmetric supersonic air jet,"** *Rev. sci. Instrum.*, May 1948, vol. 19, pp. 307–322.

A short description of the Mach-Zehnder type of interferometer and its basic theory is given, together with an example of specific application to interferogram measurements made on a 10-mm diam open air jet. Also a discussion of the details of fringe formation, fringe shift, width, orientation, density patterns, etc., is presented. Sixteen figures and illustrations are given.

The mechanical details of the four-plate interferometer used in the experiments are described, and the particular techniques of making reliable systematic optical adjustments discussed. A bright diffuse mercury laboratory arc with no diaphragms is recommended for the alignment light source. A 5461 °A green line was used (filters #3484 and #4120).

The author's experience with various other light sources and photographic apparatus is mentioned. For instantaneous photographic work a condenser discharge spark in open air between vertical magnesium electrodes was found to be the most satisfactory. The strongest spectral line was observed at 4481 °A.

The method of analyzing interferograms was illustrated by a specific example of a photographic comparison between a shadowgram and an interferogram for the 10-mm air jet operating at 60 psig tank pressure. Such items as the measurement of the photographic plates, calculation of the fringe shifts, and density are discussed. Oscillations of the fringes were revealed by 35-mm motion picture film recordings. The vertical fringes were focused in a horizontal slit near the moving film. Fringe oscillations of about 800 per sec were recorded for the above tests.

The indicated accuracy of the interferometer for density measurements for these particular tests was within 5 to 10 per cent.

Millard F. Dowell, USA

1404. **H. Schaefer, "Machmeters for high-speed flight research,"** *J. aero. Sci.*, June 1948, vol. 15, pp. 351–363.

From the two methods to measure the Mach number, making use of stagnation and free-stream pressures or temperatures, respectively, the first is chosen for the design of the Machmeter, the second being impractical because of the difficulties in obtaining the proper temperature values in flight.

The relations of stagnation and static-pressure and temperature ratio versus Mach number are derived and presented in graphs. Using the difference of these pressures as an operational quantity for the Machmeter, an altitude correction has to be applied as shown by graphs giving the relation of differential pressure versus Mach number. The present Machmeters are designed according to this principle.

The main features of the construction of a subsonic ($M = 0.3$ to 1.0) and a newly developed transonic Machmeter ($M = 0.5$ to 1.5) are shown.

A Mach Number Limit Switch is described for pilotless missiles; this special version of the Machmeter opens and closes electrical contacts controlling the fuel supply of the engine within specific limits that can be set from the outside of its case.

A sister instrument, the Mach Air-Speed Indicator, is also discussed. This is a combination of a conventional air-speed indicator and an altimeter calibrated in maximum allowable air speed for a certain Mach number; the calibration curves of this instrument are also given.

Finally the location and shape of pressure vents for transonic and supersonic air speeds are discussed.

T. v. Oosterom, Holland

1405. **C. Schmieden, "Tunnel corrections for propellers and spinning wings in a free jet and in a closed tunnel of circular cross section (Kanalkorrekturen für Luftschrauben und trudelnde Flügel im Freistrahl und geschlossenen Kanal mit Kreisquerschnitt),"** *Wiss. Veröff. T. H. Darmstadt*, Jan. 1948, vol. 1, pp. 80–92.

This report calculates the effect of the tunnel boundary (closed tunnel) or the free-stream boundary (open tunnel) on the effective angle of attack for two-bladed propellers and spinning wings. It is shown that the correction is of the same order of magnitude for the spinning wing as for the ordinary wing but is essentially

smaller for the two-bladed propeller. A comparison with the Glauert correction given in *Rep. Memo. aero. Res. Counc. Lond.*, no. 1566 is made which shows that the corrections found in the paper are of the second order compared with Glauert's, for normal conditions and not too great values of the ratio of propeller radius to tunnel (free-stream) radius.

The paper constitutes an interesting application of vortex theory. P. N. Brandt-Møller, Denmark

Thermodynamics

(See also Revs. 1359, 1376, 1381, 1389, 1421, 1424)

1406. A. W. Goldstein, S. Alpert, W. Beede, and K. Kovach, "Analysis of the performance of a jet engine from characteristics of the components. II. Interaction of the components as determined from engine operation," *Nat. adv. Comm. Aero. tech. Note*, no. 1701, Sept. 1948, pp. 1–56.

If the separate compressor and turbine performance charts of a jet engine are expressed in terms of suitable nondimensional parameters and superimposed, a useful "matching" chart is obtained. This not only shows the compressor and turbine performances at all engine operating conditions, but can also be used to predict the effect of modifications in the engine components, and to determine the acceleration surge point of the compressor. The necessary data for the construction of the charts can be obtained from a test of the complete engine with normal operating temperatures, but it may be necessary to make tests with more than a single compressor configuration to insure covering a sufficient range of turbine characteristics. Approximately correct results are possible if the data are taken from cold air calibration tests of the compressor and turbine. C. W. Smith, USA

1407. H. C. Hottel, G. C. Williams, and C. N. Satterfield, "Generalized thermodynamics of high-temperature combustion," *Trans. Amer. Soc. mech. Engrs.*, Aug. 1948, vol. 70, pp. 667–676.

In this paper the important problem of the thermodynamic calculation of combustion processes is considered in connection with the method of calculation of flame temperature and of liquid-fuel rocket-power plants. A simplified graphical method of presenting the enthalpy and entropy of equilibrium composition mixtures of carbon, hydrogen, oxygen, and nitrogen combustion products are presented at two fixed pressures. No attempt is made to compute processes involving other pressures, nor is there any estimate given of the magnitude of the errors involved in the use of the graphical data. The number of charts is decreased by the introduction of "reduced" thermodynamic functions and "fictitious" compositions. As a result, it is not easy to find, without specific calculation of cases, what magnitude of error results.
Howard W. Emmons, USA

1408. M. A. Zipkin and G. W. Lewis, Jr., "Analytical and experimental performance of an explosion-cycle combustion chamber for a jet-propulsion engine," *Nat. adv. Comm. Aero. tech. Note*, no. 1702, Sept. 1948, pp. 1–59.

The results of analytical and experimental investigations of the performance of an explosion-cycle combustion chamber for use in jet propulsion are presented. The tests were made with a combustion chamber having a timed inlet valve and several fixed area exhaust nozzles.

The experimental results show a maximum tested specific impulse of 35 lbs thrust per lb per sec air flow. (The figure for a typical turbojet engine is 60 lb per lb per sec.)

The tests show that the specific fuel consumption varies with the fuel-air ratio between four and eight lb per hr fuel per lb thrust. (The figure for a typical turbojet engine is 1.0 lb per hr per lb.) The tested maximum explosion-pressure ratio was 2.90 while the measured explosion time averaged 0.006 sec.

Theoretical analysis indicates that if the explosion time could be reduced from 0.006 sec to 0.003 sec, the specific impulse could be increased by 100 per cent and the explosion-pressure ratio could be increased by 30 per cent. C. A. Meyer, USA

1409. J. H. Burgoyne and G. Williams-Leir, "The influence of incombustible vapours on the limits of inflammability of gases and vapours in air," *Proc. roy. Soc. Lond. Ser. A*, July 21, 1948, vol. 193, pp. 525–539.

By applying an ignition flame in both quiescent and flowing gaseous mixtures in $1^7/_8$-in. and 4-in. glass tubes, the limits of inflammability were determined for various ternary mixtures consisting of a combustible gas or vapor, air, and an incombustible vapor. From these data the limiting "safe" compositions of the various mixtures were determined. The combinations studied were hydrogen, methane, ethylene, carbon monoxide, *n*-hexane, cyclohexane, and benzene with methyl bromide as the incombustible component; hydrogen and *n*-hexane with difluorodichloromethane, and methane with carbon tetrachloride. Mixtures of coal gas and air were also investigated with methyl bromide as the incombustible component, and the experimental inflammability limits checked fairly well with those computed using Le Chatelier's rule and data obtained for the various constituents.
J. Howard Childs, USA

1410. D. Bogart, D. Okrent, and L. R. Turner, "Thermodynamic charts for the computation of fuel quantity required for constant-pressure combustion with diluents," *Nat. adv. Comm. Aero. tech. Note*, no. 1655, July 1948, pp. 1–54, plus eight charts.

Charts have been developed which enable the calculation of the combustion temperature in a gas turbine when the air supply to the engine is diluted with water, alcohol, water-alcohol mixtures, liquid ammonia, liquid carbon dioxide, liquid nitrogen, or liquid oxygen for the purpose of augmenting the thrust or power output of the engine.

This report is an extension of a previous work [*Nat. adv. Comm. Aero. tech. Note*, no. 1086, 1946] which included charts for normal combustion mixtures.

The development of the eight charts is based on the assumption that the combustible mixtures are leaner than stoichiometric and that dissociation effects are unimportant. C. A. Meyer, USA

Heat Transfer; Diffusion

(See also Revs. 1394, 1433, 1438, 1439)

1411. L. B. Schueler, W. T. Reid, Paul Cohen, R. C. Corey, A. R. Mumford, C. G. R. Humphreys, and G. W. Bice, "An investigation of the variation in heat absorption in a pulverized-coal-fired water-cooled steam-boiler furnace, I, II, III, IV," *Trans. Amer. Soc. mech. Engrs.*, July 1948, vol. 70, pp. 553–614.

The four papers of this symposium report the results of tests conducted to investigate the distribution of heat flux over the walls of a specific water-cooled pulverized-coal furnace under various operating conditions. The four variables considered in the tests are; heat available, excess air, direction of fuel firing, and relative cleanliness of heat-absorbing surfaces.

The following types of test were conducted:

1 The distribution of heat flux was determined by measuring

the difference between the surface temperatures of the tube metal on the gas side and the temperature of the evaporating water in the tubes. One hundred thirty-five thermocouples were distributed over the furnace walls, each being attached to a tube at the center line (on the tube radius normal to the furnace wall).

2 Total heat absorption in the furnace was determined by measurements of energy supplied to the furnace and energy carried out in the leaving gas stream; the latter was determined by traverses of the furnace exit section for velocity, composition, and temperature of the gases. A single shielded aspiration (high-velocity) thermocouple was used for the temperature traverses.

3 One tube of the furnace wall was equipped with a Pitot tube and static pressure taps for measurement of velocity and of density variations of the fluid in the tube; these data permit computation of the heat absorbed by the fluid in the tube.

4 Four tubes were equipped with groups of thermocouples arranged circumferentially around the furnace side of the tube to determine the circumferential variation of the temperature difference between the tube surface and the evaporating water.

The measurement of heat absorption by temperature differences on the tube center lines, using flat projected wall area, showed good agreement with the measurement of total heat absorption by exit gas traverse when (1) the fluid film coefficient in the tube was assumed to be 5000 Btu/sq ft hr F, (2) the tube wall conductivity was taken as 29 Btu/ft hr F, and (3) the exit gas temperatures were corrected by Mullikin's curve for radiation error. Individual tube surface temperatures showed large periodic variations as ash accumulated and dropped off; this shows the necessity of using the average of a large number of wall temperature measurements to obtain consistent results. No account was taken of possible scale accumulations on the water side of the tubes.

The measurement of heat absorption by density variation of the evaporating fluid was not consistent with the wall temperature measurement. It was concluded that the density method is inaccurate because of differences in the steam and water velocities in the tube.

The determination of heat absorption using the circumferential average temperature difference with the developed area of the tube gave results not entirely consistent with the results using centerline temperature differences and projected area. The inconsistency was ascribed to insufficient sampling of circumferential temperatures and the effects of local ash accumulation.

The circumferential average temperature differences were compared with centerline temperature differences, as an index of wall cleanliness; the results were not considered satisfactory, again due to insufficient sampling of circumferential temperatures and local ash accumulation.

In analyzing the effect of firing rate, excess air, and firing direction upon the heat absorption it is necessary to make allowance for uncontrolled variations in wall cleanliness due to ash and slag accumulations. A method of evaluating a cleanliness factor by visual observation of the walls is presented.

In the furnace tested it was possible to shift the radiating gas mass by tilting the burners. The test results indicate that the total heat absorption is more nearly a function of the radiating mass area than of the furnace wall area. It is suggested that furnace performance should not be judged on the basis of heat available and water-cooled surface, but that the surface of the radiating gas mass should also be considered.

David A. Mooney, USA

1412. **J. Crank, "A diffusion problem in which the amount of diffusing substance is finite. IV. Solutions for small values of the time,"** *Phil. Mag.*, May 1948, vol. 39, pp. 362–376.

This paper is the fourth in a series dealing with a diffusion problem in which the amount of diffusing substance is finite. The problem considered involves diffusion into an infinite sheet of uniform thickness, and into an infinitely long circular cylinder, immersed in a solution whose concentration is always uniform. The diffusion process is supposed to take place along capillaries in the plane or cylinder, and to be accompanied by adsorption on the capillary walls. The problem is to find how the amount of solute in the sheet or cylinder changes with time.

In earlier papers solutions were given for the following situations: (1) The concentration of adsorbed solute is a linear function of the concentration of the solute free to diffuse; (2) the relation between the concentrations of adsorbed and free solute is nonlinear, and (3) questions similar to (1) and (2) for a composite circular cylinder for which the diffusion constant of the outer shell differs from that of the central core.

It turns out that the solutions worked out in the earlier papers for (1) and (2), while quite suitable for computational purposes for large values of time, are not convenient for studying what happens for small values of time. Using a procedure of S. Goldstein [*Z. angew. Math. Mech.*, 1932, vol. 12, p. 234; *Proc. Lond. math. Soc.*, 1932, vol. 34, p. 51] solutions particularly suited for computations for small values of time are worked out. A number of examples are worked out showing the relative computational advantage and disadvantages of the earlier solutions and those given in this paper. It appears from these examples that in some instances the solutions given in the present paper may even be useful in the range lying between small and large values of time.

Benjamin Epstein, USA

1413. **Marcel Michaud, "Influence of the surface condition of a radiating material on the value of emissivity at high temperature (Influence de l'état de surface d'un matérial rayonnant sur la valeur de son facteur d'émission à haute température),"** *C. R. Acad. Sci. Paris*, Feb. 16, 1948, vol. 226, pp. 568–569.

Simplified relations between the emissivity of rough and smooth surfaces of the same material are presented in this paper. It is shown, that in so far as reflectivity is dependent upon geometrical relations, the ratio of reflectivities for smooth and rough surfaces is independent of the temperature at a particular wave length. Some experimental data in support of this are presented.

Robert Bromberg, USA

(Editor's Note: A three-page translation of this article has been made by Mr. Bromberg; the editor can supply photostats at the usual rate.)

1414. **W. Traupel, "Transient heat conduction processes in plates, cylinders, and spheres (Instationäre Wärmeleitungsvorgänge in Platten, Zylindern, und Kugeln),"** *Schweiz. Arch.*, July 1948, vol. 14, pp. 193–205.

A procedure is given for computing the way in which the temperature and total heat flow change with time for a flat plate, a cylinder, or a sphere surrounded by a medium capable of transmitting heat. It is assumed that the surrounded body is of constant temperature throughout at time $t = 0$, and that the temperature of the surrounding medium is a given function of time. It is further assumed that the coefficient of heat transfer between the outer surface of the body and the surrounding medium as well as the density and conductivity of the body are all constant. The results are a direct consequence of Duhamel's superposition principle.

Benjamin Epstein, USA

Theoretical and Experimental Methods

(See also Revs. 1307, 1308, 1312, 1317, 1319)

1415. W. J. Duncan, "Technique of the step-by-step integration of ordinary differential equations," *Coll. Aero. Cranfield Rep.*, no. 4, Feb. 1947, pp. 1–23.

In Part I the author considers the step-by-step numerical integration of a system of first order ordinary differential equations over an interval, the values of the dependent variables being assigned at the initial point, the interval being divided into n equal parts. The index of such a process at any step is defined as the lowest power of n^{-1} appearing in an expansion of the accumulated error in powers of n^{-1}. The Euler method of tangents thus has index 1 at the end of the interval, the Range-Kutta method has index 4. Given the index, if the computations are carried through using a given method for two values of n, a first order correction to the obtained values can be simply obtained by dropping higher order terms in the error series in n^{-1} and computing the first coefficient. With more values of n, higher order corrections are found.

Several examples are given. Part II contains a classification and brief review of methods of numerical integration in general.

H. J. Greenberg, USA

1416. A. L. Loorye, "The reduction of round-off errors with increase of the number of measurements" (in Russian), *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, July-Aug. 1947, vol. 11, pp. 489–492.

The author makes the deliberately paradoxical claim: "If the precision of a single measurement decreases, the mean measurement error tends toward zero." Actually the following is proved: Let ξ be the deviation of the actual from the ideal pointer position and let $f(x, h)dx$ be the probability of the inequality $x \leq \xi \leq x + dx$. If $f(x, h)$ is equal to $f(-x, h)$ and decreases with increasing x, and if

$$\int^l f(x, h)dx \longrightarrow 0 \text{ when } h \longrightarrow 0 \ldots\ldots\ldots\ldots (1)$$

for a certain fixed l, the mean measurement error tends toward zero with h. This reviewer believes that the assumption (1) postulates that, for small h, the probability of a *moderate* ($< l$) deviation ξ is small. Since deviations are due to random causes, this implies that *any* departure of the measuring setup from the ideal will cause a sizable ($> l$) jump of the pointer. This surely requires a sensitive instrument and thus decreasing $\int_o^l xfd$ means increasing the precision of the instrument. If this interpretation is correct, the author has misstated his result which, in its correct form, is rather obvious.

The phrase "round-off" is mentioned in the title because scale readings are assumed to be multiples of the smallest scale division.

A. W. Wundheiler, USA

Acoustics

(See also Rev. 1362)

1417. Arthur A. Regier, "Effect of distance on airplane noise," *Nat. adv. Comm. Aero. tech. Note*, no. 1353, June 1947, pp. 1–19.

Good agreement with the inverse square law was obtained for measurements of the noise of an airplane flying overhead at altitudes up to 5000 ft. This agreement indicates that absorption is negligible within the frequency range of the plane noise (70 to 300 cps); this is believed to hold up to 1000 cps. Above this frequency the attenuation due to real and apparent losses is of importance.

Discussed in this paper are the magnitudes of losses due to viscosity and heat conduction, due to reaction between water vapor and oxygen gas in air, due to terrain, due to gusts and turbulence, due to variation of acoustic impedance, and due to refraction caused by temperature and wind gradients.

The question has been raised of the desirability of designing airplanes so that the noise will occur in the higher frequency ranges in order to take advantage of the greater atmospheric absorption at high frequencies. To obtain an answer the absorption characteristics, as well as those of the human ear, must be taken in account. Curves of loudness level versus distance from source have been plotted. These curves show that at low altitudes the higher frequencies are louder than the lower frequencies, while at higher altitudies the lower frequencies are the loudest. For example, the crossover point of the 3000 cps and the 500 cps curve is at 1000 ft, while that of the 3000 cps and 100 cps curve is at 7500 ft.

Robert Fehr, USA

1418. N. Utter and R. Kling, "On the sound velocity in certain hydrocarbon mixtures (Sur la vitesse du son dans certains mélanges d'hydrocarbures)," *C. R. Acad. Sci. Paris*, July 5, 1948, vol. 227, pp. 41–43.

The sound velocity measurements reported were designed to test the theory of M. R. Rao, according to which the cube root of the sound velocity in an organic liquid is inversely proportional to the molar volume, and the constant of proportionality is an additive function of composition. The velocity was measured by the optical diffraction method for frequencies from three to fifteen megacycles per second, at room temperature, and standard pressure. Results are quoted for various mixtures of hexadecane and normal pure heptane, as well as butylbenzene and 2, 2-dimethylbutane. The measured values fail to substantiate Rao's theory, though they closely fit a simpler law expressing the velocity in the mixture as a linear function of the velocities in the components with coefficients equal to the respective concentration fractions.

The authors have been able to detect a definitely measurable difference in velocity between 2, 2 and 2, 3-dimethylbutane, thus indicating that the position of the CH_3 group in the molecule is important in sound propagation. No trace of dispersion was discovered over the frequency range studied.

R. B. Lindsay, USA

1419. P. W. Huber and A. Kantrowitz, "A Device for measuring sonic velocity and compressor Mach number," *Nat. adv. Comm. Aero. tech. Note*, no. 1664, July 1948, pp. 1–12.

A device is described which can be used to measure velocity of sound in fluids, and which is especially adaptable for tests of rotating machines. This device is very useful for the determination of compressor Mach number (the ratio of blade-tip speed to inlet stagnation speed of sound), and it is claimed that this quantity can be determined with an accuracy of better than 0.5 per cent. For these measurements it is not necessary to know either the composition of the working fluid or the rotating speed.

The device consists of a Helmholtz resonator, whose volume can be varied. The instantaneous volume is measured very accurately by a mercury manometer. This resonator is excited by an electromagnetic transducer operating at machine-running frequency. The volume of the resonator is changed until resonance is obtained. Presence of this resonance is indicated by a hot-wire anemometer. For the volume reading, viscous effects in the orifice of the resonator have to be taken into account. This is done with the help of a parameter obtained from Stokes's solution of the flow near an oscillating wall.

Robert Fehr, USA

1420. C. F. **Eyring, R. J. Christensen, and R. W. Raitt, "Reverberation in the sea,"** *J. acoust. Soc. Amer.*, July 1948, vol. 20, pp. 462–475.

When a pulse of sound is projected into the sea from an underwater transducer, a persistence of sound is observed in a hydrophone placed near the transducer, lasting long after the original pulse has terminated, but decaying with time. This effect is known as underwater sound reverberation, and is presumed to be due to the scattering of the projected sound by elements of the sea surface and of the bottom and also by nonhomogeneities of various sorts present in the volume of the sea. The present paper includes an important summary of experimental research on reverberation conducted off San Diego during 1941-1942.

The scattering theory of reverberation is also presented and it is shown that under ideal conditions (that is, the absence of refraction caused by temperature gradients) the intensity of reverberation is directly proportional to the intensity of the emitted pulse and to its duration. Moreover, reverberation intensity due to volume scatterers varies inversely as the square of the range, whereas reverberation intensity due to surface scatterers varies inversely as the third power of the range.

Ideal conditions are rarely encountered at sea, but the experimental results quoted in the paper confirm the theory in certain special cases. However, they also point definitely to the existence of certain more or less horizontal high scattering layers in the ocean which show diurnal movement and are probably organic in character. Their exact nature demands further study. In deep water and at long ranges volume reverberation is the most important. At shorter ranges and particularly when high winds prevail, surface reverberation from the bottom can be the important factor. At long ranges, irreversible absorption by the water interferes with the simple theory to cause a more rapid decay in reverberation.

The paper presents values of volume, surface, and bottom scattering coefficients over a wide range of conditions. The description of the measuring equipment and methods is adequate. The paper should stimulate further study of the scattering of sound in the sea.

R. B. Lindsay, USA

Ballistics, Detonics (Explosions)

(See also Revs. 1381, 1384, 1385)

1421. S. S. **Penner, "Qualitative experimental verification of the change of burning rate of rocket powders with radiation path length,"** *J. appl. Phys.*, June 1948, vol. 19, pp. 511–513.

It is noted that the radiation theory of the effect of radiation path length on the average burning rate of solid fuel rocket propellants indicates a small increase [Penner, *J. appl. Phys.*, 1948, vol. 19, p. 392] in the latter, with increase in the former. Experimental studies of two types of step-machined powder grains of two to five steps were made in an attempt to verify the theory. Scatter of the experimental points was so great as to preclude quantitative conclusions, but qualitatively the theory is found to be verified.

William Pell, USA

1422. F. R. **Gantmacher and L. M. Levin, "Equations of motion of a rocket"** (in Russian), *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, May-June, 1947, vol. 11, pp. 301–312.

There is enough occasional confusion about the correct equations of rocket motion to justify the meticulousness of this paper. The "whole rocket" consists of the invariable "rocket body" S, the powder, and the gas between the exit plane and the skin. The six equations of momentum and moment of momentum should be written as if the whole rocket suddenly solidified, the moments being taken about the instantaneous mass-center (in spite of its shifting relative to S). The habitual forces and moments acting are those owing to: (1) gravity, (2) air action, (3) outside jet pressure, (4) "pure" jet action, (5) Coriolis effect, and (6) accelerations of internal gas particles relative to S.

Pure jet action is owing to the *relative* (to S) momentum escaping with the gas particles. The Coriolis vector force and moments are (not in the paper's notation!)

$$J = 2(k_p^e - k_g^e - \int (\partial \rho/\partial t)v^e d\tau), \quad H = 1_p^e - 1_g^e - \omega \times \Lambda^r - \int (\partial\rho/\partial t) r \times v^e d\tau$$

where ρ is the gas density, $d\tau$ the volume element, r the position vector, v^e the entrained velocity of a particle, ω the angular vector velocity of S, Λ^r the moment of gas momentum relative to S, k^e and 1^e the *entrained* (by S) vector momentum and moment of momentum of the powder and the gas, according to the subscript. These expressions assume practical forms for $\partial\rho/\partial t = 0$ and axial symmetry.

The paper also derives the equations of motion of the shifting mass-center (not to be confused with the motion of the mass-center of S) and those of the angular motion (in Euler's form) for shifting central axes of inertia.

The reviewer notes that the change in the moments of inertia is reflected in the Coriolis effect and in the effect of the relative acceleration of the inner gas particles. A. W. Wundheiler, USA

1423. **R. G. Stoner and W. Bleakney, "The attenuation of spherical shock waves in air,"** *J. appl. Phys.*, July 1948, vol. 19, pp. 670–678.

The peak pressure of spherical shock waves from small explosive charges has been determined as a function of distance by measuring the velocity of propagation. The charges were composed of TNT or pentolite, weighed from 1.45 to 8 lb, and were spherical or cylindrical in shape, except for one series of small rectangular blocks.

Experimental arrangements are described in detail The pressure-distance relations for the four principal charge types are given by the following fitted equations, in which π represents excess peak pressure in atmospheres, and the distance, scaled according to charge weight, is given by the nondimensional variable $Z = R/(\rho r)^{1/3}$ where R is the distance from a charge of volume r and specific gravity ρ

$^1/_2$-lb rectangular blocks, TNT

$$\pi_Q = 13.50/Z - 769.9/Z^2 + 36280/Z^3$$

8-lb cylinders, Pentolite

$$\pi_A = 10.49/Z - 135.5/Z^2 + 21070/Z^3$$

4-lb cylinders, TNT

$$\pi_B = 11.34/Z - 185.9/Z^2 + 19210/Z^3$$

3.75-lb spheres, Pentolite

$$\pi_C = 8.63/Z + 295.1/Z^2 + 7823/Z^3$$

These equations are valid for values of Z between approximately 18 and 110. The indicated probable error of the fitted curves is of the order of one per cent for intermediate distances, increasing to from two to seven per cent at the extremes of the distance range covered.

The curve for spherical charges is in agreement with the Kirkwood-Brinkley theory, and the results for cylinders having various length/diameter ratios indicate large dependence of the pressure on charge shape, at least for values of Z up to 50.

Stuart R. Brinkley, Jr., USA

1424. H. Muraour and G. Aunis, "Comparison of observed and calculated pressures of a gas mixture at elevated temperature and pressure (Comparaison des pressions expérimentales et calculées pour un mélange gazeux à haute température et sous haute pression)," *C. R. Acad. Sci. Paris*, May 19, 1948, vol. 226, pp. 1588–1590.

Experimental and theoretical pressures produced by a "hot" double-base powder fired in a closed vessel at densities of loading from 0.1 to 0.25 grams per cu cm are compared. Experimental results are from Burlot (1924) and the authors (pre-1939), recorded on crusher gages, and transformed to true pressures by comparison with piezoelectric gages. Correction is made for cooling. Theoretical pressures were computed with dissociation and Sarrau's assumption of a constant covolume. There is agreement to within about one per cent. J. Corner, England

Soil Mechanics; Seepage

(See also Rev. 1340)

1425. M. Bonneau, "Equilibrium limit and failure of continuous media (Equilibre limite et rupture des milieux continus)," *Ann. Ponts Chauss.*, 1947, vol. 117, Sept.-Oct., pp. 609–653; Nov.-Dec., pp. 769–801.

A complete review of the theory of plastic deformation of continuous media is presented, supplemented by problems from the author's experience, with particular emphasis on equilibrium and failure of soil configurations. The paper begins with a statement of the present status of the mathematical theory of the equilibrium limit in the form of a questionnaire. This is followed by a brief historical note on the development of the theory of large deformations.

Certain mathematical considerations are discussed, including singular points and lines of slip, the equations of equilibrium along slip lines and lines of constant pressure, and the role of cohesion. Hencky's theory is discussed briefly as well as the solution of Riesmann of the general problem. Certain other methods of solving the equations in special cases are presented.

Some of the specific problems discussed include the reaction to load of level ground and embankments, the problem of foundations, rupture along a circular arc in an embankment, and the relationship between Luder's lines and the rupture surface. Mathematical analogy is pointed out between Luder's lines and the Lorentz transformation.

Following a discussion of the three-dimensional problem, consideration is given to methods of calculation including the values of the coefficient for abutment loads; drilling; the two and three-dimensional problems of underground cavities, including straight tunnels; compartmented quays. The paper concludes with a brief résumé of the work of German authors, and a survey of published values of certain coefficients.

Morton B. Millenson, USA

1426. E. E. De Beer, "Data concerning the resistance in shear deduced from penetration tests (Données concernant la résistance au cisaillement déduites des essais de pénétration en profondeur)," *Géotechnique Lond.*, June 1948, vol. 1, pp. 22–39.

The paper attempts to establish a procedure for determining the shear strength of soils from a penetration test, using theoretical formulas. The development of these formulas is not given, the reader being referred to a paper in the issue of May 1, 1946 of *Génie Civil*. This makes the understanding of the arguments rather difficult. It is claimed that the shear strength determined from the penetration test is always on the safe side.

A. M. Freudenthal, USA

1427. Claude Coviaux, Application of electronics to the study of soils using the seismic method (L'application des procédés électroniques à l'étude des sols par la méthode sismique)," *Génie civ.*, June 1, 1948, vol. 126, pp. 208–210.

The seismic method is based on the measurement of wave amplitudes transmitted by the vibration of soil. The author suggests a new application of this method using electronics. The interference between waves propagated internally and along the surface results in wave maxima and minima whose positions are characteristic of certain kinds of subsoil. Thus the stratification of subsoils and the depth of the strata might be determined. The wave motion is measured by electronic apparatus (oscilloscope, reverser, low-frequency generator).

The method is also applicable to laboratory testing of the effect of the nature, density, and humidity of subsoil test specimens on the velocity of wave propagation.

Ch. Széchy, Hungary

1428. Donald W. Taylor, "Fundamentals of soil mechanics," John Wiley & Sons, Inc., New York, 1948. Cloth, 5.6 × 8.5 in. 700 pp., 239 figs., $6.00.

The main topics covered in this book are soil classification, subsurface investigations, one-dimensional consolidation, elastic theory methods for estimating earth stresses, earth settlement, shear strength and analysis, stability of slopes, lateral pressures, dam foundations, soil bearing capacity, and pile foundations.

As the title indicates, the author has laid special emphasis on fundamentals, hence the book is best suited as a student text. Nearly all equations are developed from basic principles and complete derivations included. This is particularly true in the treatment of permeability, seepage, consolidation, earth stresses, and lateral pressures.

The chapter on Stability of Slopes presents in considerable detail analyses of slopes by the circular-arc method of Fellenius, the friction-circle method, and the Culmann method, together with charts of stability numbers for certain cases. Complete submergence, the instantaneous complete drawdown case, the steady seepage case, and the case of zero resultant boundary neutral force are treated individually.

The use of flow nets in the analysis of seepage problems and the various methods which may be used for determining them are outlined. The stability of retaining walls and sheet piling under various types of back fill is discussed from the point of view of the Rankine Theory and the Coulomb Theory. Information on size and depth effects in the design of footings, and a comparison of the theoretical determination with building code specifications are contained in the chapter on bearing capacity. The last chapter is devoted to pile foundations.

Eben Vey, USA

1429. R. R. Minikin, "Retaining walls of flexible sheeting," *Civ. Engng. Lond.*, Aug. 1948, vol. 43, pp. 396–399.

Making use of a remark of Terzaghi, that in vertical-sided trenches the slipping wedge of soil is bounded by a logarithmic spiral surface which cuts the horizontal top surface of the soil at a distance equal to 0.4 to 0.5 of the height of the wedge, the author gives a simplified method for calculating retaining walls of flexible sheeting. By means of diagrams he shows that it makes very little difference if the rupture surface is assumed to be a plane, and on this basis he develops formulas and graphical constructions for computing the earth pressure, the depth of penetration of the sheet piling, and the reaction of the anchorage.

Aurel A. Beleș, Rumania

1430. **V. A. Klyachko, "On the influence of the form of grains on the velocity of the filtering stream"** (in Russian), *Notes Acad. Sci. USSR* (*Doklady Akad. Nauk SSSR*), June 11, 1948, vol. 60, pp. 1329–1332.

This paper is based on the conventional assumptions that (*a*) percolation through a porous medium occurs as if the fluid were moving through a bunch of capillary tubes under the action of a hydraulic head identical for all tubes, and (*b*) all pores of the given medium are filled with moving fluid.

The discharge velocity is proportional to the coefficient of shape Φ of the average grain defined as the ratio of its volume to its surface times the diameter of the equivalent sphere. The value of Φ for the sphere is $^1/_6$. It has been shown experimentally that if a sphere of a given weight falls through quiet water with a velocity u_1, and a grain of another shape, but of the same material and weight falls through quiet water at the same temperature with a velocity u_2, the value of the coefficient of shape $\Phi = (^1/_6)(u_2/u_1)$. Slight correction of this formula permits its use when the particle of a shape different from a sphere has a different weight.

The experiments were made with one natural sand and three artificially prepared materials using grains of a weight close to 1 mg, the porosity of the medium varying from 42.6 to 44.5 per cent. The value of the head over the surface of the filtering layer (500 mm thick) was 1000 mm.

The reviewer comments that care is recommended in applying the statements of this paper to such materials as clays with very small scalelike particles unless additional experiments extend the validity of these statements to such materials.

D. P. Krynine, USA

Geophysics, Meteorology, Oceanography

(See also Revs. 1377, 1427)

1431. **G. E. Forsythe, "Speed of propagation of atmospheric waves with changing shape,"** *J. Met.*, Apr. 1947, vol. 4, pp. 67–69.

The most discussed wave solutions of the vorticity equation have been those that assume no change in shape. The present paper omits this restriction, and derives a frequency equation directly from the vorticity equation and a definition of the speed of propagation. Its direct applicability to forecasting appears questionable due to the restrictions imposed by the initial assumptions.

Joanne Starr Malkus, USA

1432. **E. Palmén, "On the distribution of temperature and wind in the upper westerlies,"** *J. Met.*, Feb. 1948, vol. 5, pp. 20–27.

Studies of large-scale features of the belt of middle-latitude westerly winds and their relation to the general circulation of the atmosphere have, up until recently, been concerned with averages, due to scarcity of data. Daily synoptic work was thus restricted to the study of smaller areas and vertical layers, probably too limited to reveal the significant relationships. Since the war, however, the increasing abundance of upper-level observations embracing large parts of the northern hemisphere have provided opportunities for significant large-scale synoptic studies such as the present one.

One of the first discoveries so made was that the belt of high-level westerlies generally appears as a surprisingly narrow band of strong winds, embedded in relatively stagnant air masses to the south and north. The paper under review presents an analysis of a typical case of the well-developed westerly jet stream, and by means of an extended meridional cross section, synoptic maps at the surface and 500 mb (millibars), and a latitudinal wind and temperature profile at 200 mb, reveals several important facts regarding the fields of motion and temperature, and their relation.

First, it is shown that the anticyclonic latitudinal wind shear south of the maximum west wind is so strong that zones of dynamic instability may be present (because of the difficulty in accurate determination of the required values of the geostrophic wind, this question cannot yet be answered conclusively). Second, it is demonstrated that this anticyclonic shear in the warm air above the polar front may be great enough to modify the classically expected cyclonic wind shear in a frontal zone. Only fronts with solenoidal fields which are strong compared to the shear in the warm air above them will show the cyclonic shear.

Finally, the latitudinal temperature field in the vertically more stable stratosphere (studied by 200-mb temperature profiles) is offered as a tool for studying large-scale vertical motions. As an important example, the zone of low temperature to the north of the westerly jet indicates a cross-stream vertical circulation superimposed on the zonal motion. The 200-mb profiles also reveal an extremely cold (−70 C) winter arctic stratosphere.

Joanne Starr Malkus, USA

1433. **O. G. Sutton, "Convection in the atmosphere near the ground,"** *Quart. J. roy. met. Soc.*, Jan. 1948, vol. 74, pp. 13–30.

An analysis is made of the convective transport of heat through the lower atmosphere on a clear day in summer. The analysis is based on the concept of the heat exchange coefficient defined as $K_H = w'l_H$, where w' is the upward vertical air velocity of air mass as it travels the distance l_H and l_H is the "mixing length" for heat transfer. An expression for the velocity w' is obtained from the balance between the rate of energy dissipation by breakdown of the heated air mass into smaller eddies, and the rate of loss of potential energy of the heated air mass.

Utilizing the derived expression for w' and assuming constant upward heat flux, it is shown that all the quantities entering the problem of heat transfer by natural convection should be simple power functions of l_H. Since l_H is shown to be a simple power function of the height above the ground Z, these quantities are therefore simple power functions of Z.

Experimental support for the analysis is obtained from the data of Johnson and Heywood. A value of K_H of approximately 10^4 sq cm per sec was obtained at a height of 6.8 meters. As predicted by the analysis, the experimental value of K_H was found to vary with the 1.8 power of Z. A derivation leading to the temperature distribution in the lower atmosphere, resulting from the periodic rise of heated masses of air is also presented. The author concludes that: (1) On clear summer days the transport of heat in the lower atmosphere is largely by free convection; and (2) such transport is in accordance to the author's analysis.

R. C. Martinelli, USA

1434. **H. R. Seiwell, "Evaluation of sea-surface roughness from underwater-pressure recordings,"** *Trans. Amer. geophys. Un.*, Apr. 1948, vol. 29, pp. 197–206.

This is a preliminary account of two series of records of underwater-pressure fluctuations: 639 records were taken for 20 min every four hours near Woods Hole at a depth of 75 ft and 1022 records for 20 min every two hours at Bermuda at a depth of 120 ft. From the records, taken at depth h, the wave height ΔH_h corresponding to the observed pressure variation and the wave period T are measured; the wave height ΔH_s at the sea surface is then derived from

$$\Delta H_s = 1.35\ \Delta H_h \cos h(4\pi^2 h/gT^2)$$

where the factor 1.35 has been determined experimentally at both sites by direct comparison of simultaneous near-bottom and surface-wave heights.

W. R. Dean, England

1435. E. Palmén and K. M. Nagler, "An analysis of the wind and temperature distribution in the free atmosphere over North America in a case of approximately westerly flow," *J. Met.*, Apr. 1948, vol. 5, pp. 58–64.

A case of approximately westerly upper flow is chosen to construct an average meridional cross section, and so to fill in the lacunae and remove the errors which might arise in the study of single meridional cross sections. This conscientiously applied averaging process corroborates earlier findings by the senior author [E. Palmén, "On the distribution of temperature and wind in the upper westerlies," *J. Met.*, Feb. 1948, vol. 5, pp. 20–27] regarding the jet stream in the upper westerlies. The mean cross section shows (*a*) the strong high-level jet, found approximately above the frontal region at 500 mb; (*b*) a multiple tropopause; (*c*) a band of cool air to the south, and warm air to the north, of the maximum wind at 200 mb, giving an indication of the field of vertical motion; (*d*) a narrow zone (300-km width) just south of the jet showing probable dynamic instability; (*e*) an extremely cold aretic winter stratosphere. Such a period of relatively undisturbed westerly flow is suitable for a clear presentation of the features associated with the high-level west wind maximum. Joanne Starr Malkus, USA

1436. R. G. Fleagle, "The fields of temperature, pressure, and three-dimensional motion in selected weather situations," *J. Met.*, Dec. 1947, vol. 4, pp. 165–185.

These calculations of vertical motions represent only qualitatively rather large time and space averages. The results of the three examples studied, a polar anticyclone, a Colorado low, and an East Coastal cyclogenesis, although valuable for empirical forecasting, may not prove as useful as hoped in understanding the significant relations among atmospheric fields of temperature, pressure, and three-dimensional motion.

Joanne Starr Malkus, USA

1437. R. Brard, "Waves generated by a pulsating source moving horizontally with a uniform rectilinear motion (Vagues engendrées par une source pulsatoire en mouvement horizontal rectiligne uniforme)," *C. R. Acad. Sci. Paris*, June 28, 1948, vol. 226, pp. 2124–2125.

This note presents a theoretical study of waves in a fluid generated by a pulsating source moving horizontally with a uniform rectilinear motion. It is assumed that the fluid has one free surface. The principle of stationary phase is used to interpret the velocity potential generated by the source. The results are expressed in terms of a parameter whose value is proportional to the ratio of the linear velocity of the source and its period of pulsation.

An interesting application of the general theory developed to the pitching motion of a moving ship is discussed.

Louis A. Pipes, USA

Lubrication; Bearings; Wear

1438. J. Baudry, "Conditions of stability of the thermal equilibrium of a journal bearing operating in mixed lubrication region (Conditions de stabilité de l'équilibre thermique d'un palier lisse fonctionnant en régime de graissage onctueux)," *C. R. Acad. Sci. Paris*, June 14, 1948, vol. 226, pp. 1954–1955.

The author discusses the thermal behavior of a lubricated journal bearing in the region of mixed hydrodynamic and boundary lubrication (graissage onctueux) where the friction coefficient increases as the speed decreases. He assumes a rate of heat dissipation for the bearing which is a sum of two terms, one proportional to the first power and the other to the fourth power of the temperature rise.

From experimental measurements of the friction coefficient on a particular bearing in the mixed lubrication region, and the known law of its variation in the hydrodynamic region, curves are plotted of the rates of heat production and of heat dissipation against temperature. These curves are found to have generally two points of intersection, corresponding to two temperatures of thermal equilibrium. Depending on the relative slopes of the two curves these points will be stable or unstable. The one at the lower temperature will be stable and may lie in either the hydrodynamic or the mixed lubrication region. The one at the higher temperature will lie in the mixed region and will be unstable. If this latter point is reached during operation due to external causes the safe limit of operation of the bearing has been exceeded and seizure may result. John T. Burwell, Jr., USA

1439. D. Clayton and M. J. Wilkie, "Temperature distribution in the bush of a journal bearing," *Engineering*, July 16, 1948, vol. 166, pp. 49–52.

The authors report an experimental investigation of the temperature distribution in the bushing of a unidirectionally loaded sleeve bearing with forced-feed lubrication. Traverses of thermocouples along 14 axially drilled holes spaced around the bushing 0.035 in. from the bearing surface were used, thus obtaining temperature measurements in both the circumferential and axial direction. By means of additional measurements in a second set of axial holes located farther from the bearing surface, extrapolation of the temperature to the surface was made and plotted.

The variation of the temperature distribution with oil-supply pressure, oil-inlet temperature, journal speed, and method of oil supply is reported. The methods of oil supply included both holes and short axial grooves located opposite the load direction and 90 deg on either side of it, and also a circumferential groove extending over the unloaded half of the bushing.

An attempt was also made to determine the maximum temperature in the oil film itself by dispersing finely shredded particles of a graded series of low-melting-point metals in the oil and observing with a microscope any change from jagged to globule form thus indicating that the melting point had been exceeded.

John T. Burwell, Jr., USA

Marine Engineering Problems

(See also Rev. 1437)

1440. E. C. Goldsworthy, "The development of cycloidal propulsion with particular reference to Voith-Schneider," *Trans. Instn. Engrs. Shipb. Scot.*, July 1948, vol. 91, pp. 425–448.

This paper presents a discussion of the application and experience of the Voith-Schneider and other cycloidal propellers in which the blades projecting perpendicularly from the surface of the hull are rotated in a cylindrical path about an axis also perpendicular to the hull. The system offers advantages in maneuvering since, by changing the angles of attack of the blades as they rotate, any desired amount of thrust within the design limits can be exerted in any chosen direction. Some 250 ships have been fitted with propellers working on this principle with 2200 hp as the maximum in one unit. F. E. Reed, USA